Acta Mathematica Nitriensia
Vol. 8, No. 1, p. 28 – 38
ISSN 2453-6083

Solving Geometric Tasks by 9th Grade Pupils and Using GeoGebra Software

Veronika Bočková [a *] – Gabriela Pavlovičová [a]

[a*] *Department of Mathematics, Faculty of Natural Science and Informatics, Constantine the Philosopher University in Nitra, Tr. A. Hlinku 1, SK-949 74 Nitra*

Received April 19, 2022; received in revised form April 21, 2022; accepted April 23, 2022

Abstract

Geometry is one of the most important branches of mathematics education in Slovakia and abroad. According to our experiences, pupils often have difficulties solving various geometric tasks. It can be caused by a lack of pupils´ knowledge of geometry or the low levels of their mathematical competencies in geometry. For these reasons, the research focuses on evaluating six geometric tasks solutions at various levels of mathematical competencies. The research confirmed our assumptions that 9th-grade pupils have significant problems with solving geometric tasks at the higher level of mathematic competencies. There are a lot of opportunities to support the development the mathematical competencies in geometry. One of the possibilities for it is to use various dynamic geometric software. Therefore, after evaluating the geometric tasks, there are offered the solutions of geometric tasks by using the GeoGebra software in the article.

Keywords: mathematical competencies in geometry, geometric tasks, nine-grade pupils, GeoGebra software

Classification: D40, D60, G10, N80

Introduction

Geometry is an essential part of mathematical education, and it has an important place in the Slovak mathematics curriculum. This fact is confirmed by twelve thematic units, which deal with geometry at the lower secondary education. Table 1 shows all thematic units the pupils would pass from fifth to ninth grade at elementary school. The content standards determine the minimum content of education of individual thematic units. The performance standard formulated by active verbs determines the minimum level of the acquired curriculum. The specific performance and content standards of individual thematic units dealing with geometry are given by the National Education Programme (2014).

Grade	Thematic units
5.	Geometry and measurement
6.	Area of rectangle, square and right triangle Angle, size of the angle and operations with angles
7.	Volume and surface of cube and block

*Corresponding author; email: veronika.bockova@ukf.sk
DOI: 10.17846/AMN.2022.8.1.28-38

8.	Triangle, identical triangles Parallelogram and trapezoid, area of triangle Circle Volume and surface of the prism
9.	Plane symmetry (axial and central) Pythagorean theorem Similarity of triangles

Table 1: Thematic units which deal with geometry

Although many math lessons deal with geometry, pupils still have a significant problem with solving geometric tasks. As we have observed in recent years, pupils have a deficient level of geometric knowledge in Slovakia (Bočková, Pavlovičová, Čeretková, 2020). The national test indicates this trend at the second level of elementary school (Testovanie 9), which is intended for nine-grade pupils.

Figure 1 shows the percentage success rate of pupils in various fields of mathematics.

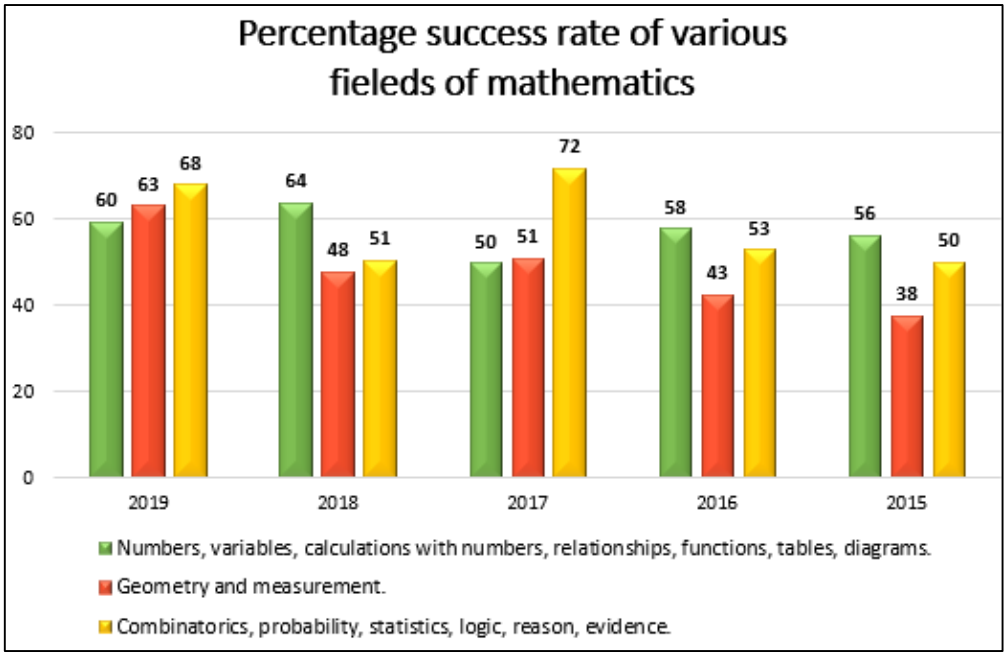


Figure 1: The percentage success rate of various fields of mathematics in Testovanie 9

We can see that pupils achieved the lowest (in years 2015, 2016, 2018) or second-lowest percentage (in years 2017, 2019) success rate in geometry in Testovanie 9 (NUCEM, 2015 – 2019). The low level of pupils´ geometric knowledge may be caused by pupils being deficient in mathematical competencies in geometry. The low level of mathematical competencies in geometry causes pupils to solve only standard tasks and have problems with solving non-standard tasks.

Mathematical competencies

The current topic is the competency development of pupils and students at all levels and types of schools. As Turek (2005) mentions, the content of the term competence is appropriately defined by the behavioral definition of Schoonover Associates (2001): *"Competence is a behavior (activity*

or complex of activities) that characterizes an excellent performance in a certain area of activity". The school mathematics is focused on developing pupils' mathematical competencies as was defined by the European Parliament. European Parliament defines mathematical competence as *"the ability to develop and apply mathematical thinking to solve various problems in everyday situations*". According to Šedivý et al. (2013), mathematical competencies "*are general mathematical knowledge, abilities and skills that correspond to the respective levels of education*."

In the literature, we can find various classifications of mathematical competencies. OECD PISA (2010) has suggested eight distinctive mathematical competencies that are relevant and meaningful across all education levels:

- mathematical thinking and reasoning,
- mathematical argumentation,
- mathematical communication,
- modeling,
- problem posing and solving,
- representation,
- symbols and formalism,
- aids and tools.

It is necessary to draw simultaneously upon many of those competencies when doing real mathematics. OECD PISA (2010), to operationalize these mathematical competencies divide them into three competency clusters:

- **Reproduction** - Pupils or students repeat the learned material, perform routine calculations and solve simple problems.
- **Connections** - Pupils or students can solve problems that are not routine with the use of integration, interconnection, the connection of known methods, simple supplementation of acquired knowledge and modeling.
- **Reflection** –Pupils or students can plan strategies for solving problems that are more complex and require an original mathematical approach as well as a combination of different methods.

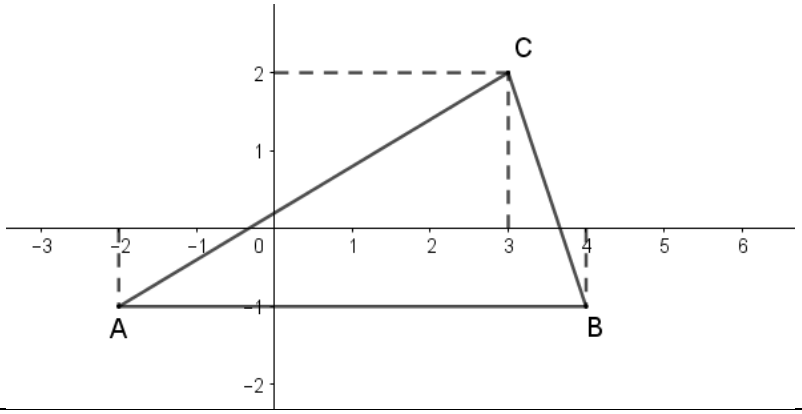
Methodology

Our research focuses on evaluating solutions to six geometric tasks at various levels of mathematical competencies. After evaluating the task, we suggest a swift, easy, and interactive solution for the tasks in GeoGebra software. The research was realized in May 2021, and the research sample consisted of 200 9^{th} grade pupils (104 boys, 96 girls) who solved six geometric tasks. Mark 2 was the average mark in pupils´ mathematics evaluation.

Pupils solved six geometric tasks: the first two (1^{st} and 2^{nd}) tasks were focused, at the reproduction level of mathematical competencies, the other two tasks (3^{rd} and 4^{th} tasks) at the connection level and the reflection level's last two tasks (5^{th} and 6^{th} tasks). The following tasks are analyzed and evaluated:

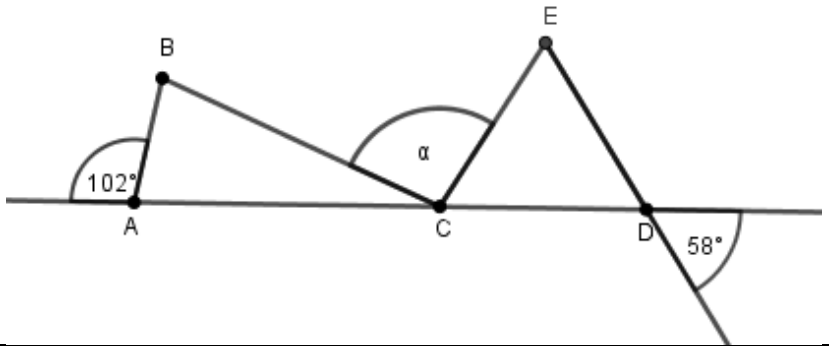
1. *If the area of a rectangle is 48 cm^2 and the length of the rectangle is 6 cm, what is the width of the rectangle?*
2. *The arc of circle c with a central angle $\alpha = 25°$ has a length of 62.8 cm. Determine the circumference of the whole circle c.*

3. *The triangle ABC is drawn in the rectangular coordinate system. Determinate how many square units is the area of a triangle.*

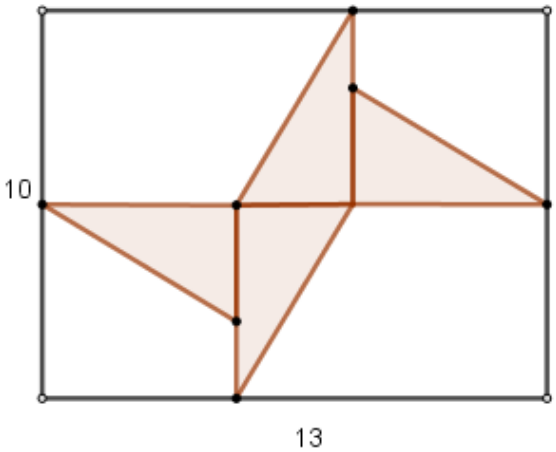


4. *The lengths of the sides of the triangle ABC are three natural numbers. The two shorter sides have lengths: c = 6 cm and b = 8 cm. Determine the length of the third side so that the perimeter of the triangle is as large as possible.*

5. *In the picture are two isosceles triangles - triangle ABC with base AB and triangle CDE with CD base. Determine the size of the angle α.*



6. *A pattern is composed of four identical triangles in the rectangular tile. Determine the percentage representation of this pattern.*



Results

We observed that some pupils solved all geometric tasks without any problems in the solutions. All the tasks were solved correctly by 6.5 % of pupils, and 9.5 % of pupils solved only one task incorrectly. On the other hand, many pupils had problems with solving geometric tasks. 41% of pupils solved only one geometric task and 11% did not solve any task.

The content analysis of incorrect solutions of the first two geometric tasks at the reproductive level shows that pupils often do not know the formulas for calculating the area of geometric shapes and the formula for calculating the length of a circular arc. In the solutions, we observe that pupils also have problems determining the circumference of shape, which they must put into the correct formula. Analyzing the incorrect solutions of the other two tasks at the connection level shows that many pupils cannot link their knowledge and solve the tasks correctly. Pupils know the formula for calculating the area of geometric shapes and the necessary mathematical relationships. However, they do not know how to obtain additional information from the task to solve it. In the solutions of the last two tasks at the level of reflection, we observe that pupils can´t plan a comprehensive approach to their solution. They often only calculate the individual parts of the tasks that they cannot subsequently connect. We also observe numerical errors and inattention errors in solving all the tasks.

Figure 2 presents the achievements of pupils in solving geometric tasks. As we can see, the first geometric task was the easiest for the pupils (85% of pupils could solve the task, 11.5 % of pupils made incorrect solutions, and 3.5 % of pupils did not start solving the task). On the other hand, the last task was the most difficult for them (only 20 % of pupils could solve the task, 29 % made incorrect solutions, and 49 % did not start solving the task). From the solutions of the tasks, we see that the success of solving geometric tasks decreases with the increasing level of mathematical competencies needed to solve the task. Figure 2 also suggests that pupils have difficulties solving tasks at the higher levels of mathematical competencies.

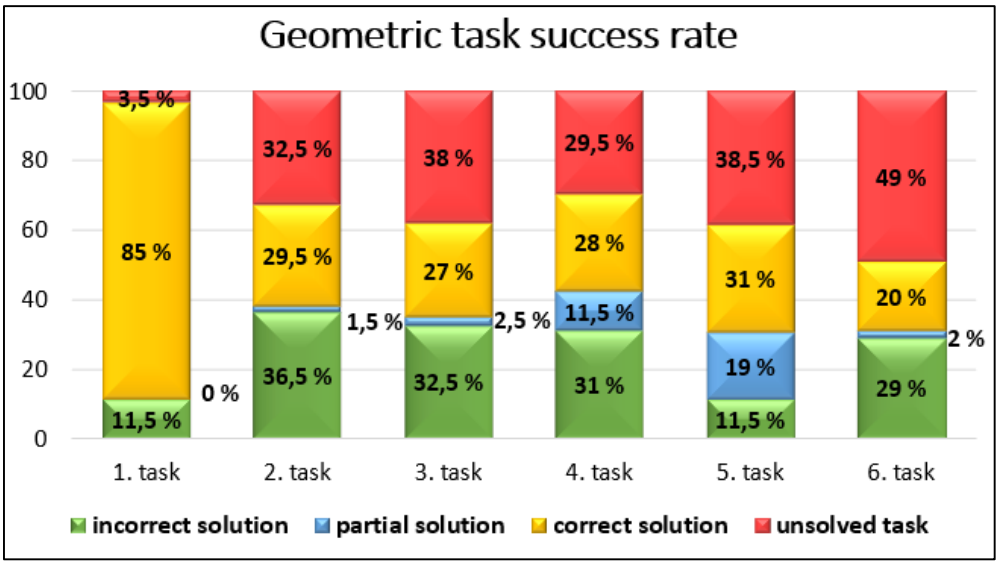


Figure 2: Geometric task success rate according to the correct, incorrect, partial solution or unsolved task

Solution of the tasks using GeoGebra software

In the article, we have shown that pupils have significant problems with solving geometric tasks. Pupils used paper, pencil, and calculator while solving the geometric tasks. It is questionable whether pupils would achieve more correct solutions using GeoGebra software. GeoGebra offers them various options for calculating given geometric tasks and eliminates various numerical errors or errors of inattention. In the following section, we offer the solution to the given tasks using GeoGebra.

1st geometric task

We design the first task in GeoGebra so that the pupils can move with the slider and determine the correct width of the rectangle (Figure 3).

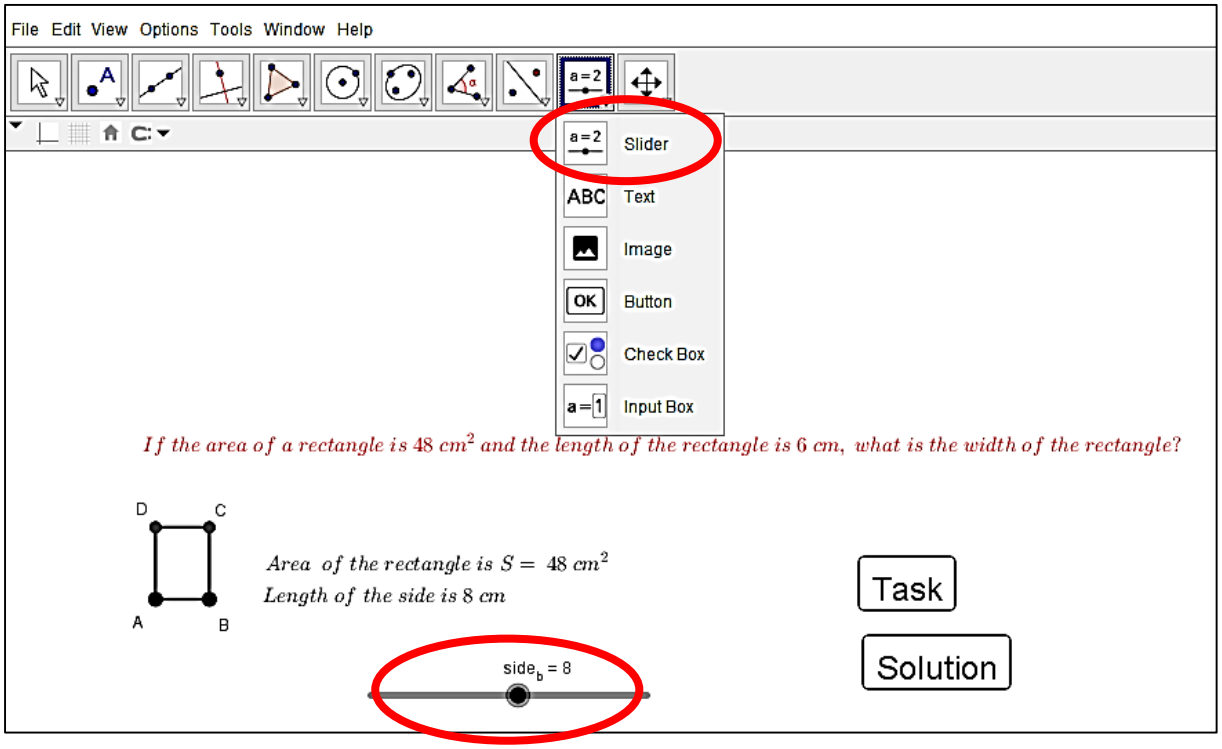


Figure 3: The solution to the 1st geometric task

The task and solution of the 1st geometric task are available on the GeoGebra website:

https://www.geogebra.org/m/bz4bu9yp

2nd geometric task

The task has a straightforward and quick solution in GeoGebra. Pupils can use the GeoGebra tool – Distance and Length, and they determine the length of the circle (Figure 4).

The task and solution of the 2nd geometric task are available on the GeoGebra website:

https://www.geogebra.org/m/madrwdzy

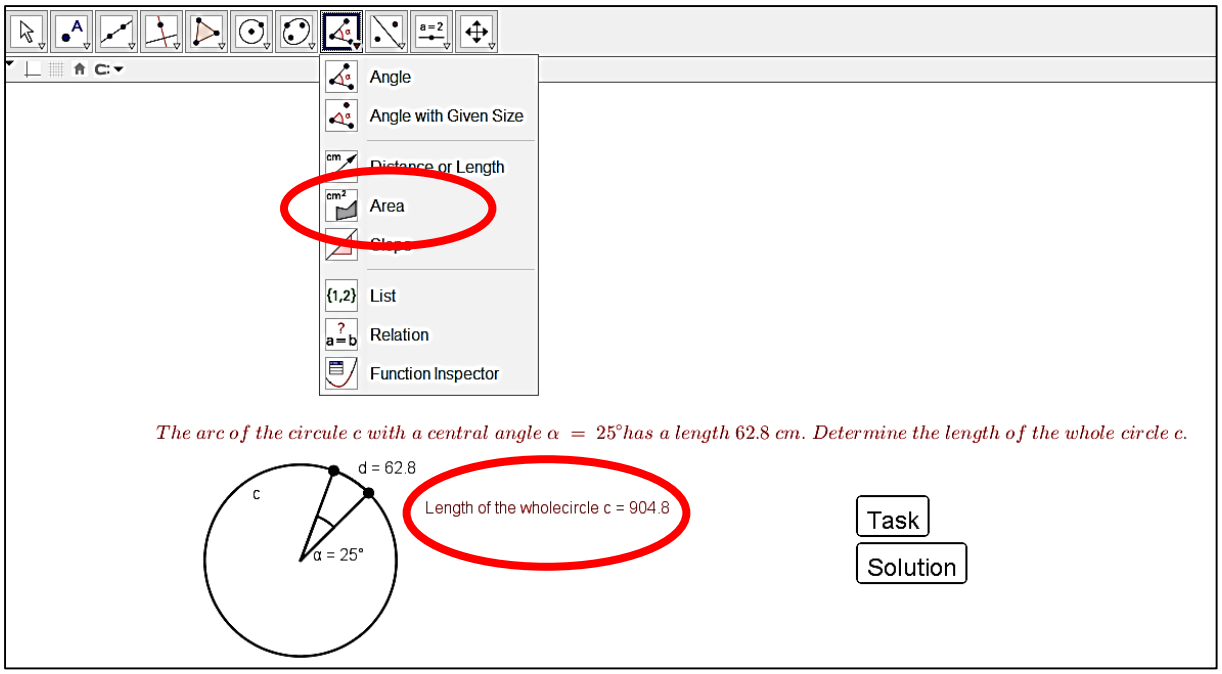


Figure 4: The solution to the 2nd geometric task

3rd geometric task

The task has two different solutions:

The first solution is similar to the previous solution for the second task. Pupils can use the GeoGebra tool – Area, and they determine the area of the given polygon - triangle *ABC* (Figure 5).

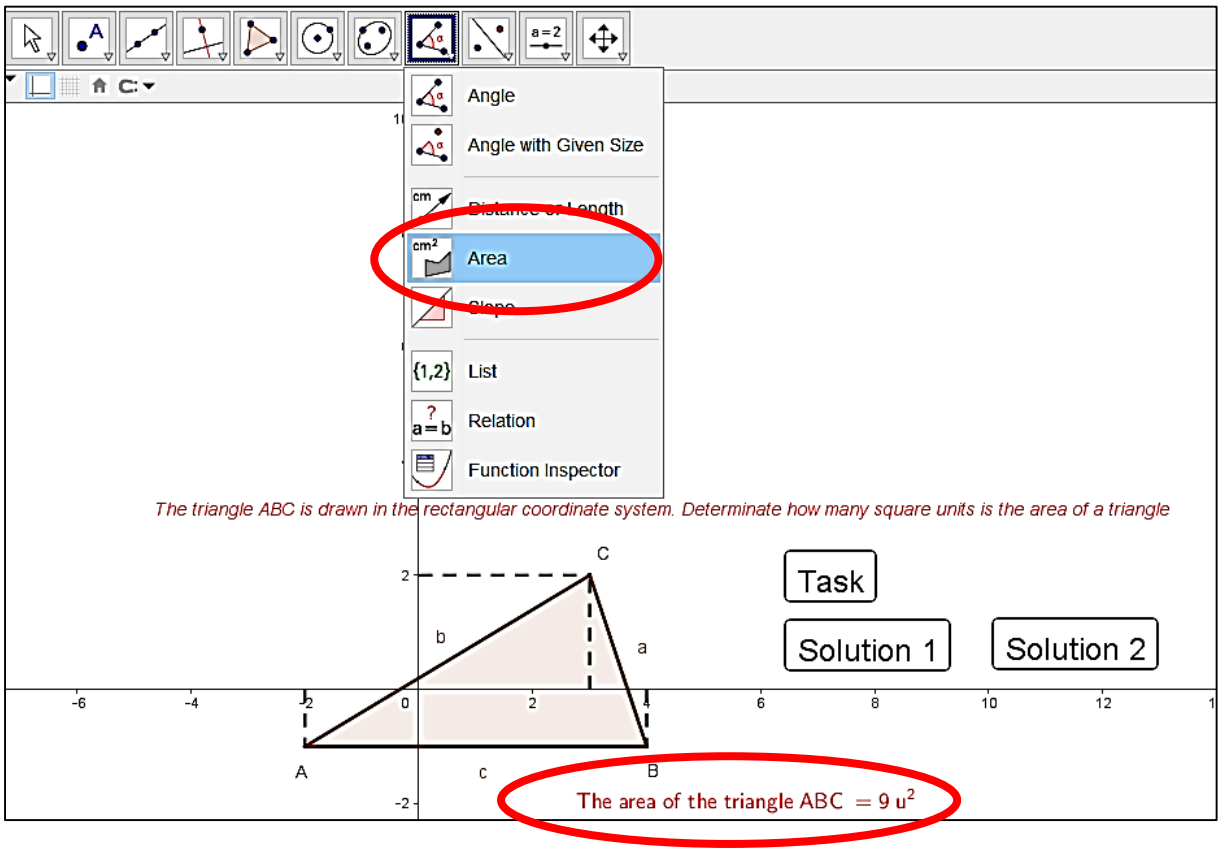


Figure 5: The solution to the 3rd geometric task

The second solution is focused on determining the length of the side *c* and the length of the altitude to the side c. Pupils mark the given line segments, and then the length of the side c and length of the altitude to the side *c* is displayed automatically in the algebra window or in the picture. If pupils know the lengths of the side *c* and the altitude to the side *c*, they can use a formula to calculate the area of the triangle. This solution eliminates the most common mistake of pupils - incorrect determination of the side length and altitude length.

The task and solution of the 3rd geometric task are available on the GeoGebra website:

https://www.geogebra.org/m/puvtwebn

4th geometric task

We design the fourth task in GeoGebra so that the pupils can move with the point *C* and determine the correct and the most significant third side of the triangle. (Figure 6).

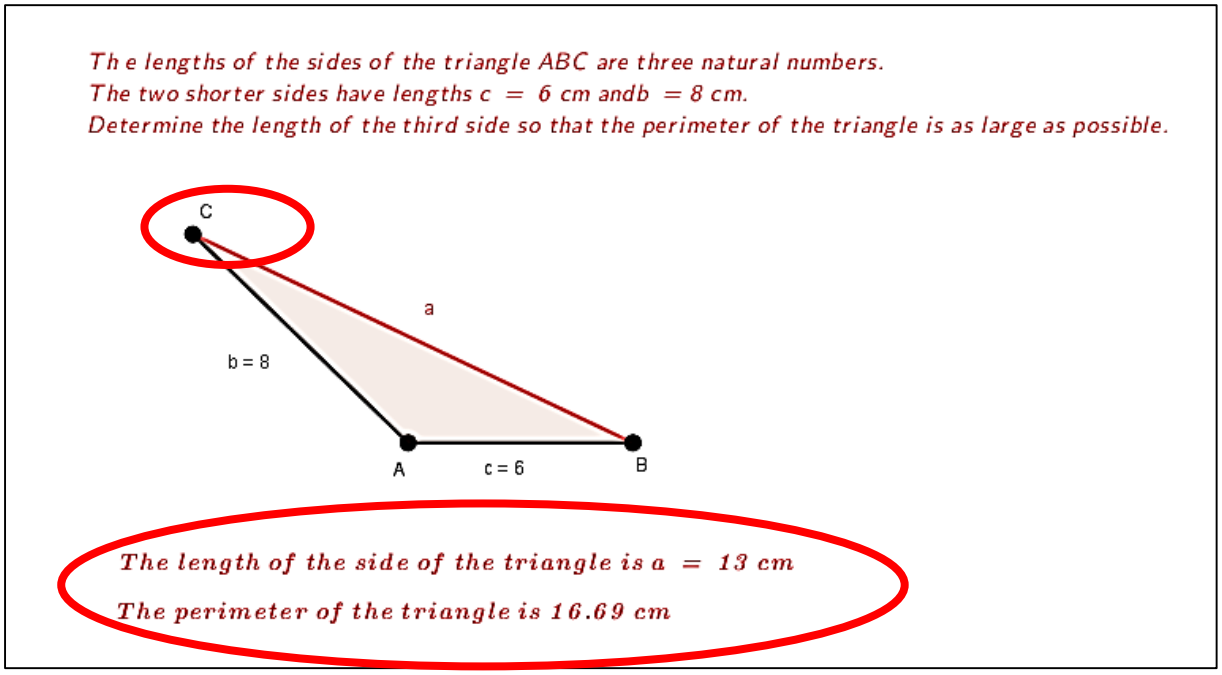


Figure 6: The solution to the 4th geometric task

The task and solution of the 4th geometric task are available on the GeoGebra website:

https://www.geogebra.org/m/ebckggkp

5th geometric task

The task has a very simple and quick solution in GeoGebra, too. Pupils can use the GeoGebra tool – Angle, and they determine the size of the angle *BCE* (Figure 7). Pupils mark the given points *B, C, E* (*B, E* are points at the arms of the angle, the point *C* is its vertex) and the size of the angle is displayed automatically in the algebra window or the picture. We suggested some hints for the pupils.

These hints can eliminate the most common mistake of pupils – they did not know which angles in the isosceles triangles were the same.

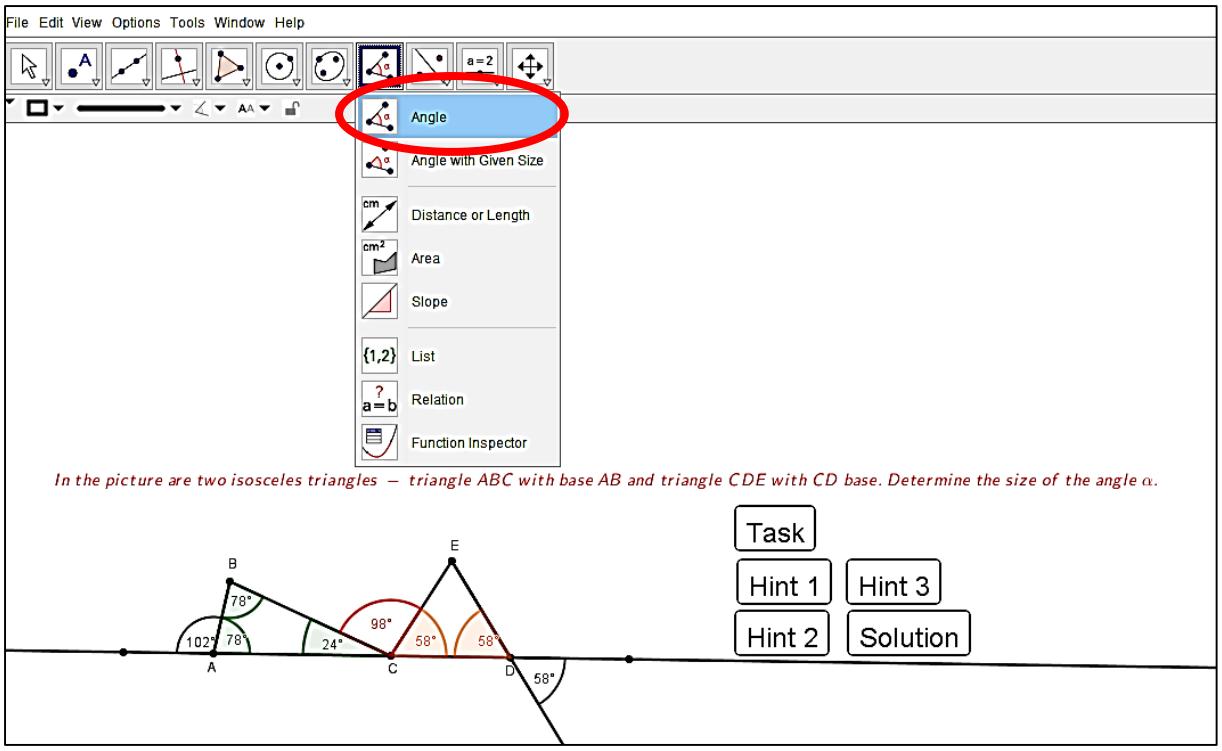


Figure 7: The solution to the 5th geometric task

The task and solution of the 5th geometric task are available on the GeoGebra website:

https://www.geogebra.org/m/k7fswncq

6th geometric task

Pupils solve the task in four steps by GeoGebra:

- They determine the area of one identical triangle by the tool Area.
- Pupils calculate the area of the whole pattern – multiply the area of one triangle by four.
- They determine the area of the rectangular tile by the tool Area.
- Pupils determine the percentage of the pattern – they write a command, which calculates the percentages, and they use GeoGebra as a calculator.

If pupils have a problem with the solution of the task, they can help with our solution. They can play the solution in four steps in GeoGebra (Figure 8). In our experience, if students correctly calculate the tile's content, they correctly calculate the percentages of the pattern.

The task and solution of the 6th geometric task are available on the GeoGebra website:

https://www.geogebra.org/m/ukt4dqyk

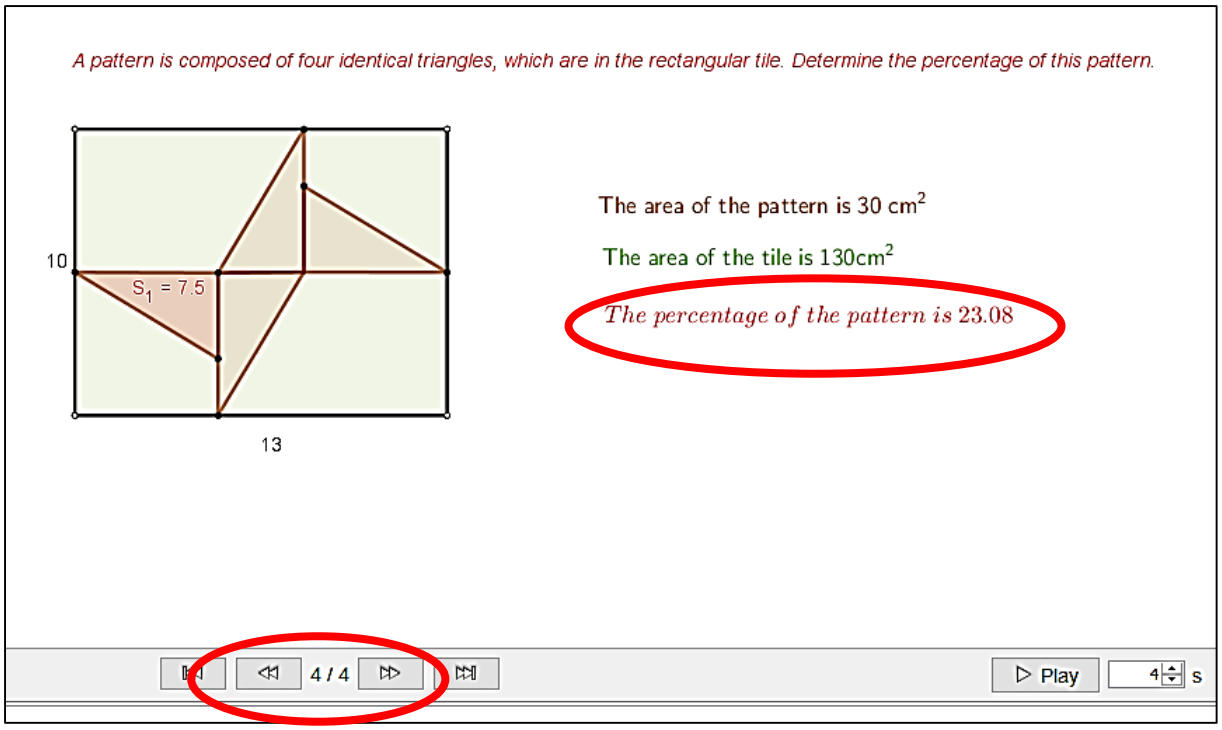


Figure 8: The solution to the 6th geometric task

Conclusion

The research was focused on the analysis of solving six geometric tasks by nine-grade pupils at the second level of primary education.

The research results show that pupils have a lot of problems with the solution of geometric tasks. The pupils solved only one task from six with more than 50 percent success. They had problems with solving tasks that required higher mathematical competencies skills. The right solution for these tasks made less than 31 percent of pupils. Our findings are in line with the results of other researchers and the results of Slovak national tests. Our research also confirms that the problem of understanding geometry is long-term.

One way to promote understanding of the geometry curriculum is to use pedagogical software in geometry lessons. According to Kalaš (2013), pedagogical software helps students to experiment, manipulate with objects, discover relationships and patterns, or construct them. Special cases of pedagogical software offer the pupils a wide range of different construction tools and also provide many tools for examining geometric objects that can be manipulated within the program.

Some studies as Kutluca (2013) or Abdullah (2015) confirm that the use of dynamic geometry software in teaching improves understanding of the geometry curriculum. Pupils which were learned by classical methods achieved a lower level of geometric thinking than pupils which were learned with the effective use of dynamic geometric software after learning the same topics from geometry. Their knowledge was also more permanent.

We believe that using dynamic geometric software, for example, GeoGebra, or other new methods in the teaching - learning process can help pupils develop geometric thinking and mathematical competencies in geometry.

Acknowledgment

This paper was created with support of project ***KEGA: 019 - UKF – 4/2020 -*** *Podnetné didaktické postupy vo vyučovaní zobrazovacích metód v sekundárnom matematickom vzdelávaní s ohľadom na požiadavky spoločnosti a praxe.*

References

[1] Štátny pedagogický ústav. 2014. *Matematika a práca s informáciami*. [online]. Bratislava: Štátny pedagogický ústav, 2014. [cited 15.2.2022]. Retrieved from: https://www.statpedu.sk/files/articles/dokumenty/inovovanystatnyvzdelavaciprogram/matematika_nsv_2014.pdf .

[2] Bočková, V., Pavlovičová, G., Čeretková, S. 2020. Increasing pupils' interest in geometry through mathematical trails. In *ICERI 2020: 13th International Conference of Education, Research and Innovation*. Spain, Seville: IATED Academy, 2020. pp. 2038 – 2047.

[3] NUCEM. 2015 – 2019. *Testovanie 9 - Priebeh, výsledky a analýzy, years 2015 – 2019*. [online]. Bratislava: Národný ústav certifikovaných meraní vzdelávania. 2015 - 2019. [cited 15.2.2022]. Retrieved from: https://www.nucem.sk/sk/merania/narodne-merania/testovanie-9

[4] Turek, I. 2005. *Inovácie v didaktike*. Slovakia, Bratislava: MPC, 2005. 358 p. ISBN 80-8052-230-8.

[5] Schoonover Associates. 2001. *Competency Q&A*. [online]. Retrieved from: http://www.schoonover.com/resource Center/Q-A.htm .

[6] Šedivý, O. et al. 2013. *Vybrané kapitoly z didaktiky matematiky.* Slovakia, Nitra: Univerzita Konštantína Filozofa, 2013. 220 p. ISBN: 978-80-558-0438-5.

[7] OECD. 2010. Main Features of the PISA Mathematics Theoretical Framework. In: *Learning Mathematics for Life: A Perspective from PISA*. France, Paris: OECD Publishing, 2010.

[8] Kalaš, I. et al. 2013. *Premeny školy v digitálnom veku*. Slovakia, Bratislava: SPN, 2013. 256 p. ISBN 9788010024094.

[9] Kutluca, T. 2013. The Effect of Geometry Instruction with Dynamic Geometry Software; GeoGebra on Van Hiele Geometry Understanding Levels of Students. In *Educational Research and Reviews.* 2013, 8(7), pp. 1509 – 1518.

[10] Abdullah, A. H. et al. 2015. Enhancing Students' Geometrical Thinking Levels through Van Hiele's Phase-based Geometer's Sketchpad-aided Learning. In *2015 IEEE 7th International Conference on Engineering Education*. New York, USA: Curran Associates, Inc., 2015. ISBN 978-4799-8811-2. pp. 106 – 111.