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Teacher trainees crossing the barrier between the concrete and the abstract - (mis)understanding of geometric concepts

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Abstract

In general, abstraction and correct understanding of fundamental concepts play an important role in teaching practice. A good teacher not only has to know the proper meaning of concepts, but also be able to use them correctly and recognize situations in which it is sensible to use particular terms. Pupils acquire notions of terms in their early childhood and then in primary educational level, therefore primary teachers' correct understanding of the concepts is essential. During their university studies some teacher trainees find out that they have incorrect or insufficient notion of geometric terms. This can be remedied in several ways. Useful remedy methods can be discussions, Socratic dialogue or use of various material teaching aids. We managed to interlink these three methods in a 90-minute lesson of an optional course covered in study plan of the Teacher Training Master Programme for Primary Education at Constantine the Philosopher University in Nitra, Slovakia. The students who participated in the lesson were in their first or second year of the master degree study. The submitted contribution comprises a brief description of the lesson and a summary of findings we obtained by means of worksheets filled in by the teacher trainees.

Keywords: polyhedron, base of a solid figure, skeleton models of solid figures, Socratic dialogue

Classification: D60, D70

Introduction

All their lives people learn mainly from their experience. For children it is natural to touch and try everything. After having started their school attendance, they are forced to learn more and more through reading and writing. For many children this change might be very hard. Teachers can soften the blow of this transition by means of suitable aids (Gabajová & Vankúš, 2011). Also, it is very important for children to learn to express themselves correctly from the very beginning of their schooling. Correct expressing includes use of appropriate register and terms for particular concepts. In this sense the teacher is a key character from whom learners adopt language devices and register. That is why we considered it a good idea to make the primary teacher trainees construct skeleton models of solid figures from easily available drinking straws.

Objectives and research tools

The research was conducted in a lesson from an optional mathematics course with 50 students at the age of 22 and over who were in their first or second year of Teacher Training

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Master Programme for Primary Education. The lesson contents were focused on both educational and research goals. The educational goals included revision or acquisition of students' knowledge of regular polyhedrons, planar figures which form the faces of given solids, number of edges, faces and vertices, and last but not least, correct labelling of the solid and planar figures. An additional objective was to introduce a simple way of preparing a teaching aid which the teacher trainees might find useful in their teaching practice later on. The research goals included investigating the teacher trainees' deficiencies in understanding selected geometric concepts, and verifying applicability of Socratic dialogue within mathematics education of pre-service teachers. The research tool for data collection was a two-page worksheet, with a task to define selected geometric concepts on one page, and a task to fill in a table related to selected solid figures on the other page. The worksheets filled in by the teacher trainees were subjected to content analysis.

Polyhedrons in mathematics education

According to the National Educational Programme ISCED 1 pupils first encounter solid figures in the fourth year of primary school, when the primary focus is put on cubes, cube constructions and drawing their plans and elevations (ISCED 1). However, pupils encounter many other solid figures in their natural environment. Thus, they can also encounter regular polyhedrons. It is crucial that the teacher can correctly label the fundamental parts of the solids, so that their pupils are exposed to correct register properly describing the concepts. What children acquire at this stage, they (mis)use later on in their lives.

Material teaching aids

One of the basic principles in mathematics instruction, and in education in general, is the principle of clearness and illustration. This principle highlights the importance of concept formation by means of illustrations and demonstrations, and the importance of knowledge and skills acquisition and habit formation through sensual perception of phenomena and objects, such as drawings, photographs, objects, and also inner experience and images elicited by narration. This can be efficiently provided by material teaching aids (Šedivý, 2006). A teaching aid is a material object that directly embodies particular information which pupils are expected to learn. Teaching aids can convey the informative contents via certain technical device or directly (Driensky & Hrmo, 2004). According to Gábor (1989) there are three main types of teaching aids, namely (i) demonstrative and frontal, (ii) audio-visual, (iii) and material aids.

In the analysed lesson tactile teaching aids were used. Despite they might be marked by technical imperfections and properties of the material, they play an important role in the first stages of abstraction which leads to formation of concepts about elementary geometric terms (Vallo, Rumanová, Vidermanová, Barcíková, 2013). In the lesson teacher trainees were provided with material for construction of the skeleton models of solids which allowed them to work with the models immediately. Teacher trainees could also find that helpful when filling in the table in the worksheet (Fig. 3).

Lesson structure

Two groups of teacher trainees attended two 90-minute long lessons (1 lesson per group) with the same contents and structure, and the total number of the teacher trainees was 50. Each of the lessons consisted of two main parts. The first part of the lesson was focused on the teacher trainees understanding of selected geometric concepts. Students were asked to define the following seven geometric terms in their own words – planar figure, solid figure, vertex, side, edge, face, and base (basal face). The results obtained from analysis of that lesson phase are discussed in (Vitézová, Naštická, 2015).

During the activity students had difficulties to express their thoughts and grasp their notions of the geometric concepts. We believe that in order to eliminate this problem it would be beneficial to discuss the topic with students more often, talk with them about what particular terms mean, and when it makes sense to use some terms (see *Socratic dialogue* below). For instance, such geometric terms and concepts are base (face), base (side) and the like. During the lesson several students defined the base of a solid as *that face on which the solid is standing*. Such understanding of the term, however, is wrong, since the solid figure, e. g. a pyramid, can "be standing" also on one of the side faces. Moreover, in the abstract geometric theory an expression like "be standing on something" seems ridiculous, pointless, irrelevant, and out of the context. Therefore it is necessary to emphasize the discussion and its importance in acquisition of correct geometric concepts.

The next part of the lesson was focused on construction of skeleton models of solid figures and acquisition of knowledge about solids, especially regular prisms, pyramids and polyhedrons. Students were asked to construct skeleton models of selected solids with the use of a string and drinking straws. The selected solids were namely the five Platonic solids (regular tetrahedron, octahedron, dodecahedron, icosahedron, cube), right-regular hexagonal prism and pyramid, right-regular pentagonal prism and pyramid, right-regular four-sided pyramid and right-regular three-sided prism. The model of icosahedron had already been constructed by the teachers before the lesson as this solid might seem too demanding and time-consuming for the teacher trainees. Students, working in threes or fours, managed to construct all the remaining ten solids (Fig. 1), despite minor difficulties which they successfully and independently resolved.

Figure 1

As a follow-up activity, students were asked to fill in a table related to the solids models of which they had just constructed. They were asked to state the names of the solids, number of faces, edges and vertices in the solids, names of the planar figures which form the faces of the solids, and determine if the solids have bases, and how many.

In this contribution the analysis focuses primarily on results related to Platonic solids (Fig. 2), i. e. convex regular polyhedrons whose faces are mutually congruent regular polygons, and whose vertices are incident with constant number of edges or faces. Any Platonic solid can be circumscribed with a sphere.

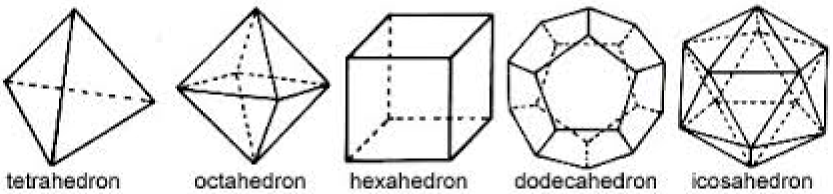

Figure 2

Figure 3 shows a correctly filled-in table. In this contribution the analysis is primarily focused on solids No. 1, 2, 3, 4 and 11.

	Name of solid figure	Number of faces	Number of edges	Number of vertices	What planar shapes are the faces of this solid?	Does this solid have any bases? If yes, how many?
1.	Cube	6	12	8	Square	No
2.	Regular tetrahedron	4	6	4	Equilateral triangle	No
3.	Regular octahedron	8	12	6	Equilateral triangle	No
4.	Regular dodecahedron	12	30	20	Regular pentagon	No
5.	Right-regular hexagonal prism	8	18	12	Regular hexagon, rectangle/square	Yes, 2
6.	Right-regular hexagonal pyramid	7	12	7	Regular hexagon, isosceles triangle	Yes, 1
7.	Right-regular pentagonal prism	7	15	10	Regular pentagon, rectangle/square	Yes, 2
8.	Right-regular pentagonal pyramid	6	10	6	Regular pentagon, isosceles triangle	Yes, 1
9.	Right-regular four-sided pyramid	5	8	5	Square, isosceles triangle	Yes, 1
10.	Right-regular triangular prism	5	9	6	Equilateral triangle, rectangle/square	Yes, 2
11.	Regular icosahedron	20	30	12	Equilateral triangle	No

Figure 3

Results obtained from the tables filled-in by students

As mentioned above, the subject of analysis of student worksheets which is presented in this contribution is the part about regular polyhedrons. Our objective was to determine if teacher trainees can properly distinguish between planar and solid figures and their constituents, if they can name the geometric figures, state the numbers of edges, faces and vertices, and also what difficulties arise within this issue. We decided not to proceed with the students to investigate relations between the numbers of the constituents of the solids and develop the task to a higher level with focus on mathematical reasoning, proofs and induction, as it would very much exceed standard requirements on the primary level teachers' mathematical education.

As far as naming the polyhedrons, students had difficulties mostly with the tetrahedron. Out of all 50 students 38 managed to name it correctly. Some of the rest incorrectly referred to tetrahedron as a four-sided pyramid, or even equilateral triangle, which is a planar figure, not a solid. The rest of the polyhedrons were named correctly, except for one case when a student labelled the icosahedron as a dodecahedron, which was probably just a lapse due to lack of concentration. Similarly, one student labelled the cube as a tetrahedron.

1.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of six congruent square faces.
2.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of four congruent equilateral triangles.
3.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of eight congruent equilateral triangles. Four edges meet at each vertex of this solid.
4.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of twelve congruent regular pentagons. Three edges meet at each vertex.
5.	With the use of drinking straws and string, make a skeleton of a right-regular hexagonal prism.
6.	With the use of drinking straws and string, make a skeleton of a right-regular hexagonal pyramid.
7.	With the use of drinking straws and string, make a skeleton of a right-regular pentagonal prism.
8.	With the use of drinking straws and string, make a skeleton of a right-regular pentagonal pyramid.
9.	With the use of drinking straws and string, make a skeleton of a right-regular four-sided pyramid.
10.	With the use of drinking straws and string, make a skeleton of a right-regular triangular prism.
11.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of twenty congruent equilateral triangles. Five edges meet at each vertex.

Figure 4

Determining the number of faces of the polyhedrons caused no severe difficulties to the students. They could simply derive it from the name of the solids, and in some cases it was clear from the initial instruction according to which students constructed the skeleton models. In addition, students had the models at their disposal while filling in the table.

The situation was similar with the number of edges. An exception was the icosahedron. Only around quarter of the total number of students managed to determine correctly the number of edges in icosahedron, though, most of the students who stated incorrect number made just minor mistakes (by one or two edges more or less than the correct number was).

Next, students were asked to state the number of vertices of the solids. Students made most mistakes in case of the dodecahedron. Only half of the students stated the correct number of its vertices. One student made a minor mistake (by one vertex more than the correct number was), the rest of the students stated that the dodecahedron has 15 vertices, and it was most likely due to mistake of one of the students and the other just copied the incorrect number as they were allowed to work in groups.

The next piece of information which was asked from students in the worksheet was to name the planar figures which form the faces of the solids. Correct answers were considered also those in which students did not specify precisely the type of certain polygons, e. g. if they stated correctly that faces of icosahedron are triangles, omitting the adjective *equilateral*, the answer was assessed as correct unless they stated wrong type, e. g. scalene. In this sense none of the solids were correctly described by all students, but the maximum number of students who gave incorrect answers for one of the solids (namely the dodecahedron) was only six. In case of other polyhedrons the number of incorrectly stated answers was less than six.

What we find alarming is the difficulty some students had when labelling the pentagon, as some of them labelled it incorrectly as a "solid with five faces" (NB: in the Slovak language, which was the language of instruction in the analysed lesson, "pentagon" and "solid with five faces" are single words with the same initial syllable), and also vice versa. Similarly, some students incorrectly labelled tetrahedron as equilateral triangle, which could imply serious errors in distinguishing between planar figures and solids.

Socratic dialogue

At first glance determining the number of bases in the regular polyhedrons seemed to be very easy for the students. However, in the lesson this issue was the most discussed. In fact, without applying the Socratic dialogue, as explained below, the final answers of students would be different. After successful use of the Socratic dialogue all students realized that in case of regular polyhedrons it is not wise and useful to talk about bases.

The method of the Socratic dialogue, which we applied in the lesson, is based on the premise that learners learn more when they are engaged deeply in investigating certain issue, when they put themselves questions and gradually seek for the answers. The Socratic dialogue positively contributes to development of critical thinking and it is also useful as a diagnostic tool for determination of the level of critical thinking. It has its origin in ancient Greece where Socrates used dialogue and questions in order to discomfit people who were overly self-confident in their opinions and beliefs. We used this method in order to assist the students in their considerations and steer them to correct ideas. In the following paragraph we state a short retrospective transcription of our dialogue with the students.

T – teacher, **S** - students

T: *How many bases are there in a cube?*
S: *One*. / *Two*. / *None*. / *Six*.
T: *What do we mean by the base of a solid?*
S: *The face the solid stands on.*
T: *So, if we turn the cube around and put it on the desk with some other face down, do we change its base?*
S: --------------------
(Students remained silent as they did not know the answer or did not understand the question.)
T: *When does it make sense to consider the concept of base?*

Although students sat up, none of them attempted to reply to the last question. Therefore we asked them to recall that pupils at primary and also secondary schools label the faces of cube as left, right, front and rear face, and upper and lower base, but the purpose of it is just to facilitate the communication between the teacher and pupils. However, when talking about cube as an abstract geometric figure with certain properties the concept of base plays no role, as well as in case of the other Platonic solids, since all its faces are congruent and there is no need to use the concept of base when computing the volume of cube. On the other hand, it is useful to consider base in case of pyramids and prisms, as their base often has different shape than the other faces, and also the surface area of the base is used in computation of their volumes. This is what we wanted to draw the students' attention on.

In the first question in the dialogue we could have used any of the five Platonic solids. Cube seemed to be the most reasonable as people in general are mostly familiar with cube and its properties.

Conclusion

When students start their university studies it is assumed that they are able to independently learn various terms, concepts and theories. They should be able to manage that on various levels of abstraction and complexity. It is also assumed that they already posses high critical and logic thinking competences. For this they must have had developed cognitive competences corresponding to the stage of formal operations in terms of Piaget theory of cognitive development (Piaget, 1970). At this stage people in general are able to use their abstract thinking and formal mental processes. If these competences are not developed enough, various complications occur, since at university level geometric concepts are introduced in their abstract sense, not the concrete one. Furthermore, such concepts are then introduced as tools for further education. For instance, the concept of base is important for computation of volume of pyramids and prisms, and there are plenty of such links. This was also the case of our students – teacher trainees for primary education. Unfortunately, many of them still do not have sufficiently acquired terms and concepts and developed abstract thinking. Our multiple experiences with the analysed student group support our belief that without the presence of the concrete models of solids they would not be able to actively participate in the activities and discussions in the lesson. The above mentioned deficiencies might be eliminated by means of properly led discussion about the main issue of the lesson.

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