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Developing Combinatorial Thinking

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Abstract

The article presents remarks concerning the possibility of introducing elements of probability in teaching at the primary stage of education. In our paper we focused on combinatorial thinking. In the course of the study, the pupils were asked to solve a few problems. They included problems related to combinations (two-element and three-element combinations of a four-element set) and permutations (three- and four-element ones).

Keywords: probability, pupils' combinatorial thinking, primary education

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Introduction

Elements of geometry, arithmetic, and algebra are introduced from the beginning of mathematical education. At the same time, probability (here understood as a fusion of elements of probability theory, combinatorics and descriptive statistics) occupies very little space in the curricula. In the current Polish core curriculum in mathematics – primary education (from 4th to 6th grade of primary school), there are only some elements of descriptive statistics. The pupil is required to master the following skills: gathering and organizing data, recognising and interpreting data presented in texts, tables, diagrams, and graphs (see Core Curriculum, 2012).

Due to the fact that teaching probability is so limited, the research on the possibility of introducing some elements of stochastics from the beginning of the primary education is very interesting. Needless to say, it is not the matter of familiarizing pupils with ready-made components of probability but of providing them with opportunities of exploring the concepts of probability and developing their probabilistic thinking. These mathematical activities seem to be important for the educational process as the surrounding reality abounds with random events.

The theorists of mathematical instruction point at the necessity of familiarizing pupils with the elements of probability as early as possible. According to H. Freudenthal, "... *the process of developing students' probabilistic thinking should be carried out in a systematic way, parallel to the development of deterministic thinking. Moreover, it should be continued over many years of education and, as a consequence, it should be started as early as possible*" (cf. Łakoma. 1988). It is noteworthy that H. Freudenthal in his concept of teaching emphasises the importance of guiding the students into the process of mathematisation, which is understood as organising the reality through mathematical measures, in as diversified

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manner as possible, whereby the probability theory is regarded as one of the best ways to achieve this goal (Łakoma,1989).

Analysis of results

The research on pupils' combinatorial thinking was conducted in May 2015 (at the end of the school year) in a primary school near Krakow. It was effectuated in two stages. During the first stage, the pupils worked on a survey. The second stage of research included discussing the solutions of the problems with the pupils. The study was conducted in a small (ten-member) group of fourth-grade pupils representing various levels of mathematical skills.

The goal was an attempt to answer the question whether the fourth-grade pupils were capable, using their existing skills and knowledge, of solving combinatorial problems.

In the course of the study, the pupils were asked to solve a few problems. These included problems related to combinations (two-element and three-element combinations of a four-element set) and permutations (three- and four-element ones).

Due to the fact that the pupils were not familiar with combinatorial terms, the problems were worded as follows:

- P1. In how many ways can you invite 2 out of 4 people to a party?
- P2. In how many ways can you invite 3 out of 4 people to a party?
- P3. In how many ways can you load 3 blocks with different colours (yellow, green, and blue) onto 3 carriages so that there is exactly one block in each carriage? Write or draw all solutions.
- P4. In how many ways can you load 4 blocks with different colours (yellow, green, blue, and pink) onto 4 carriages so that there is exactly one block in each carriage? Write or draw all solutions.
- P5. How many different three-digit numbers can be composed of digits: 1, 2, and 3? Write or draw all solutions.
- P6. How many different four-digit numbers can be composed of digits: 1, 2, 3, and 4? Write or draw all solutions.

When answering the questions, pupils enjoyed full liberty to choose the method of presenting their solutions. This allowed us to observe the manners in which they mathematised the described situations.

In P1, most pupils put down all possibilities. In addition, in their solution of the problem, most of them assigned names to particular people mentioned in the problem or marked them with different colours, drawing coloured lines grouped in twos.

Almost all pupils presented solutions with a similar structure to the one depicted in Figure 1.

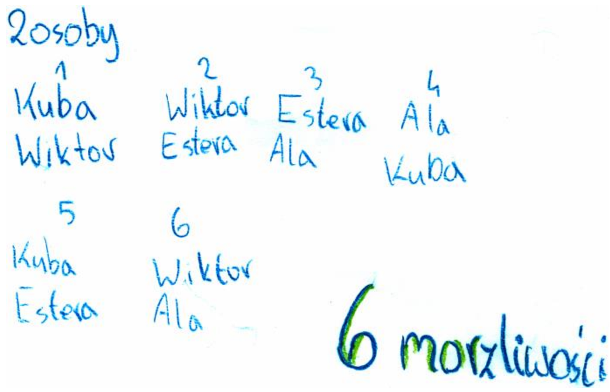

Figure 1.

Having listed all possibilities, pupils gave the answer. There are six possibilities.

One of the solutions worthy of mentioning is the one where the author obstinately tries to present the wording of the problem with the letter symbols (see Figure 2.)

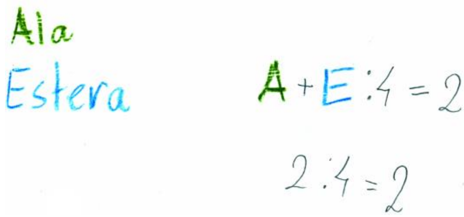

Figure 2.

The author writes the formula and then she assumes that A+E =1. This conclusion follows from the fact that the correct answer to the question posed in the problem is 2.

When solving P2, most pupils chose the identical method as in P1. Figure 3 presents a sample correct solution of P2.

Figure 3.

It is noteworthy that during the interview summing up different solutions of P2, one of the pupils pointed out that there must be four possibilities. She stated that inviting 3 people out of 4, we leave one person out, and one person may be selected in four different ways.

Answering the question of P3, most pupils obtained the correct answer (six possibilities), whereby the pupils drew all the possibilities (cf. Figure 4).

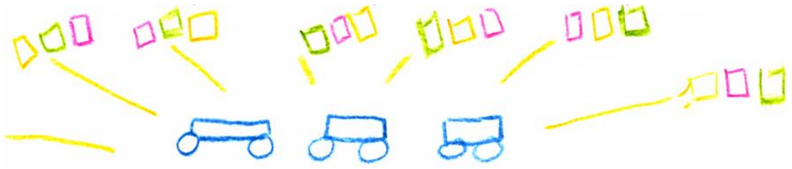

Figure 4.

Three pupils did not obtain the correct solution. Two of them only presented the data of the problem by drawing them (cf. Figure 5).

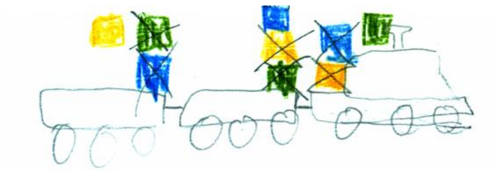

Figure 5.

The third pupil gave (drew) only three possibilities. The essence of this solution was based on the fact, which was not included in the problem's wording, that a block with a particular colour may be placed only once in a given carriage. (see Figure 6).

Figure 6.

Answering the question contained in P4, the pupils applied a method analogical to the one used in P3. Unfortunately, the correct answer was given only by one person, who started by drawing all possibilities (the pupil drew all trains with a pink block in the first carriage – six trains) and then wrote that the number of trains will be four times bigger, i.e. there will be 24 trains.

The remaining pupils made errors in their solutions, mainly consisting of omitting certain possibilities (permutations). This resulted probably from the fact that the number of all trains is as high as 24; therefore their identification (enumeration) requires a sufficiently systematic approach, which some of the pupils lacked. For illustration, two selected solutions are presented (see Fig. 7 and Fig. 8).

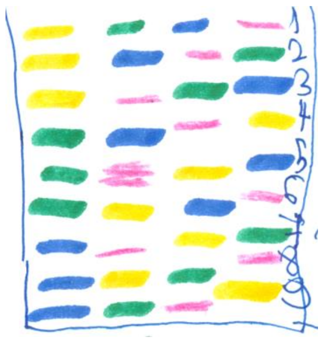

Figure 7.

Figure 8.

The mathematical layers of problems P5 and P3, as well as problems P6 and P4, are identical; therefore the numerical answers are also identical. The pairs of problems differ in their wording, whereby the problems P5 and P6 are purely mathematical. The pupils did not notice the analogy between the pairs of problems and they tried to solve each problem separately. Meanwhile, problems P5 and P6 posed much more difficulties. Perhaps this results from the fact that as the pupils solved a mathematical problem, they tried to apply familiar algorithms and courses of action, whereas when solving P3 and P4 they were guided by experience and common sense.

It is notable that in problems with realistic wording, pupils presented the possibilities in a more systematic way than in those related to numbers.

Problem P6 was solved correctly by one person only. The reasoning of this pupil was analogical to that proposed by the same person in the solution of P4 (see Figure 9).

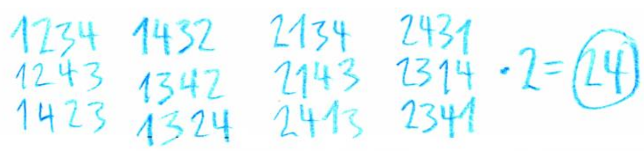

Figure 9.

Among the examined pupils, one person solved many problems with numerical answers based on estimation/guessing the number of possibilities.

Conclusion

In the final interview, the surveyed pupils said that the proposed tasks were interesting and different from those solved during the mathematics lessons on a daily basis. They also pointed out that the issues referred to in the tasks occurred in the everyday life.

The observations made during the research bring us to the conclusion that some probabilistic issues may be introduced at the beginning of the primary stage of education. With the right formulation of problems it is possible to develop the pupils' ability to encode information. Assigning tasks similar to those described above makes it possible to shape basic probabilistic intuitions as well as a systematic approach in presenting all possibilities. Therefore, it also leads to the development of logical thinking.

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