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Deficiencies in Geometric Language Register and Knowledge of Pre-service Teachers for Primary Education

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Abstract

Children adopt the language register of their close relatives and of their teachers. In terms of mathematics education, the most important language and information input is exposed to children by their teachers at primary educational level. The submitted contribution aims to investigate the deficiencies in fundamental geometric knowledge of university students within Teacher Training Master Programme for Primary Education at Constantine the Philosopher University in Nitra, Slovakia. The research was conducted in April, 2015, within a 90-minute long university seminar at the Department of Mathematics. Altogether 50 students of the master programme filled in a worksheet focused on selected fundamental geometric terms. The worksheet served as a warm-up and a follow-up activity for the main activity of the seminar when students constructed 'skeleton' models of selected solid figures. The presented results are based on observation and content analysis of the worksheets.

Keywords: prism, pyramid, skeleton of a solid figure, errors

Classification: D60, D70

Introduction

During their lifetime people learn mainly through experience. Thus, it is natural that children need to try and touch everything. After children start attending school, they have to learn more and more through reading and writing. For some children this change can be difficult. Teachers can make this transition easier for pupils, e. g. by using appropriate educational aids (Gabajová & Vankúš, 2011). In addition, it is very important that children learn to express themselves properly, already within the primary educational level. This is also related to using accurate register and terms of the specific school subjects. Teachers play crucial role in achieving this objective, since pupils adopt their language devices and register. That is why we selected the below described activities, i. e. making skeletons of solid figures with the use of drinking straws and string, for university students within Teacher Training Master Programme for Primary Education (further only 'primary teacher trainees').

Objectives and research tools

We can distinguish two types of objectives that we set to achieve by the activities. The first group consists of the educational goals. We aimed to help the teacher trainees revise their knowledge about right-regular prisms and pyramids. We focused on planar figures which

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form faces of the solid figures, and on number of edges, faces and vertices, and on names of the solids and planar shapes. Another educational goal was to introduce a simple way of creating an educational aid, which could be useful in their future teaching practice.

The research objective was to identify the errors that students made when filling in the table (see Fig. 2) and when defining several mathematical terms. The terms we selected for the activity are fundamental in geometry, and we believe primary teacher trainees should be able to explain their meanings accurately.

The tool we used for data collection is a two-page worksheet, with selected geometric terms which the teacher trainees were asked to define on one page, and a table about selected solids they were asked to fill in on the other page (Fig. 2). After the teacher trainees filled in and handed in their worksheets, the content analysis of the worksheets was carried out.

Prisms and pyramids in mathematics education

According to the National Educational Programme ISCED 1, solids are firstly introduced to pupils in the fourth year of elementary school, when the focus is primarily on cubes, building cube constructions and drawing their plans (ISCED 1). In the fifth year also cuboid is introduced, though only in a propaedeutic form; prisms are not introduced before the eighth year, and pyramids in the ninth year of elementary school (ISCED 2).

Nevertheless, it is inevitable that teachers in primary educational level use proper mathematical terms when referring to the fundamental objects, solids and their constituents, so that pupils are exposed to correct information from the very beginning. What pupils learn at the primary level is what they come with to further classes.

Material aids

One of the fundamental requirements on mathematics education is the formation of thought images and notions through visual perception and objective knowledge. In other words, it is the necessity to uphold the principle of clearness and illustration which represents the pupils' need to acquire new knowledge through concrete sensory perception of phenomena and objects. Observation of real objects, drawings and photographs, thought images elicited by speaking and listening can serve this purpose (Šedivý, 2006), as well as material educational aids for specific curriculum. Driensky and Hrmo (2004) characterize educational aids as material tools which are direct vehicles of information that pupils are supposed to acquire. An educational aid can provide educational content through a technical device or directly (Driensky & Hrmo, 2004).

Gábor et al. (1989) distinguish three main groups of educational aids: a) demonstrative and frontal, b) audio-visual, c) material. From the perspective of didactics of mathematics educational aids can be divided into two groups, according to the target group of people they are meant for, either for teachers, or for pupils.

In the activity we designed for the teacher trainees haptic (tactile) aids were used. Despite material aids are often technically inaccurate, they are very influential in the initial abstraction stage of mental ontogenesis, leading to formation of thought images representing fundamental geometric notions (Vallo, Rumanová, Vidermanová, & Barcíková, 2013). Students – teacher trainees were provided with material for production of 'skeleton' models of solid figures. After they managed to make the models, they could touch them and observe them in order to be able to fill in the table in the worksheet (Fig. 2).

Activities with students

Within the university seminar, which lasted for 90 minutes, students did two main activities. Altogether 50 primary teacher trainees took part in two such seminars. Below the activities and the results are described in more detail.

The first activity

In the initial part of the seminar students were asked to think of and write down in their own words definition of several geometric terms, namely planar figure, solid figure, vertex, side, edge, face and base of a solid figure. Students wrote their suggestions in the worksheets. Before analysing the student definitions, let us introduce definitions proposed by mathematicians.

A **planar figure** can be defined as a subset of the two-dimensional Euclidean plane which can be described with the use of points, lines and circles, sets, logic and arithmetic symbols, and the notion of distance (Hejný et al., 1989). We expected more simple formulation by students, e. g. that a planar figure is a geometric shape in a two-dimensional plane.

A **solid figure** is a part of space bounded by a surface which does not intersect itself (Pavlič, 2001). A solid is a closed three-dimensional geometric figure (Clapham & Nicholson, 2009).

A **vertex** of a polygon is any point at which two sides of the polygon meet; a vertex of a polyhedron is any point at which (at least) three edges of the polyhedron meet (Alexander & Koeberlein, 2011).

A **side** is one of the lines joining two adjacent vertices in a polygon (Clapham & Nicholson, 2009). It is a line segment which forms a part of the boundary of a planar figure.

An **edge** of a polyhedron is any line segment that joins two consecutive vertices of the polyhedron (Alexander & Koeberlein, 2011). It is the common side of two boundary polygons of a polyhedron.

A **face** of a polyhedron is any one of the polygons that lies in a plane determined by the vertices of the polyhedron; it includes base(s) and lateral faces of prisms and pyramids (Alexander & Koeberlein, 2011). It is a boundary polygon of the polyhedron.

The **base** of a solid figure is the face of a solid figure to which an altitude is drawn (Alexander & Koeberlein, 2011).

As we have already mentioned, we did not expect such accurate definitions of the notions. Students were asked to explain the notions in their own words. We were aware of the fact that it is very difficult to be accurate in definitions. Also experts in didactics of mathematics warn that it is rare to find such a fundamental mathematical term which could be given an absolutely exact definition (Fischer, Malle, & Bürger, 1992). It is said that accuracy of mathematical language is relative (Hejný, et al., 1988).

Expected errors

According to Hejný et al. (1990) there are three types of bond disruption within the process of notion formation, namely: (i) wrong thought images are assigned to words and symbols, (ii) no thought images are assigned to words and symbols, (iii) no language expression has developed (within the individual brain) for thoughts and thought images.

We expected that in the solutions of our students all the three types would occur, and that the most frequent errors would be (i) and (iii). Our expectations proved to be correct.

Results

As we could foresee, all involved students described the base of a solid as “the part of the solid which it stands on”. They did not care much about why and when (for what purposes) the notion of base is necessary in mathematics.

We further expected that students would define a planar figure as “a shape lying in a plane, or drawn on the paper”. However, some of the student definitions were positively surprising, such as “a planar figure is a set of points in a plane, for example a triangle” or “it is a shape in a plane, determined by at least three line segments which meet at three points”.

Most of the students described a solid figure as a “three-dimensional shape with certain volume and surface area”. Some students also stated that “it is a closed region of the space”, and that “unlike a planar figure, a solid figure is in the space and is observed in 3D version”.

Students described a vertex as “the point at which two or more lines meet”, or “the place where two or more line segments intersect each other”. These and other similar student definitions of vertex met our expectations.

When asking students for their definitions of terms *side*, *edge* and *face,* our main focus was on determining if they knew when it is appropriate to talk about sides and when about edges, and what is the main difference between them. Several students stated that a side “is a line segment joining vertices of a shape”, yet, many students did not state if the shape is two- or three-dimensional. Some students referred to a side as a line, or they related it to both planar figures and solids (in the Slovak language, *side* is used only when talking about planar figures, *edge* and *face* only in relation to solids). For instance, “a side is the line connecting two boundary points of a solid or a planar shape”, or “it is a line which forms the shape, for example in a square all four sides are perpendicular to each other; it is related to planar figures”.

As for the term *edge,* several students stated quite accurate definitions, such as “an edge is a line segment, bounded by two vertices, connecting two adjacent faces; it is related to solid figures”. A less accurate, but revealing much valuable information about the learner perception, is the following definition of an edge, stating that “it is a line at which as if two faces were broken”.

Attempting to define the notion of *face,* students stated that “it is a planar shape which bounds the perimeter of the solid”, or that “it is a set of points in a plane, which is bounded by the edges of the solid; faces are planar figures which the solid consists of”.

The educational goal of this activity was to make students aware of the main differences and relations between sides, edges and faces. As evidenced above, several students understand the notions and managed to explain them using their language register quite accurately. Yet, many of the students do not really understand the notions, and thus cannot use them properly.

The second activity

In the second part of the seminar students worked in twos or threes. Their task was to construct ‘skeleton models’ of eleven solids with the use of drinking straws and string. The selected solids were regular tetrahedron, octahedron, dodecahedron, cube, right-regular hexagonal prism and pyramid, right-regular pentagonal prism and pyramid, right-regular four-sided pyramid, right-regular triangular prism, and regular icosahedron. Students were not asked to construct the model of icosahedron, since it would take them too much time. Except for icosahedrons, students managed to construct models of all ten solids (Fig. 1).

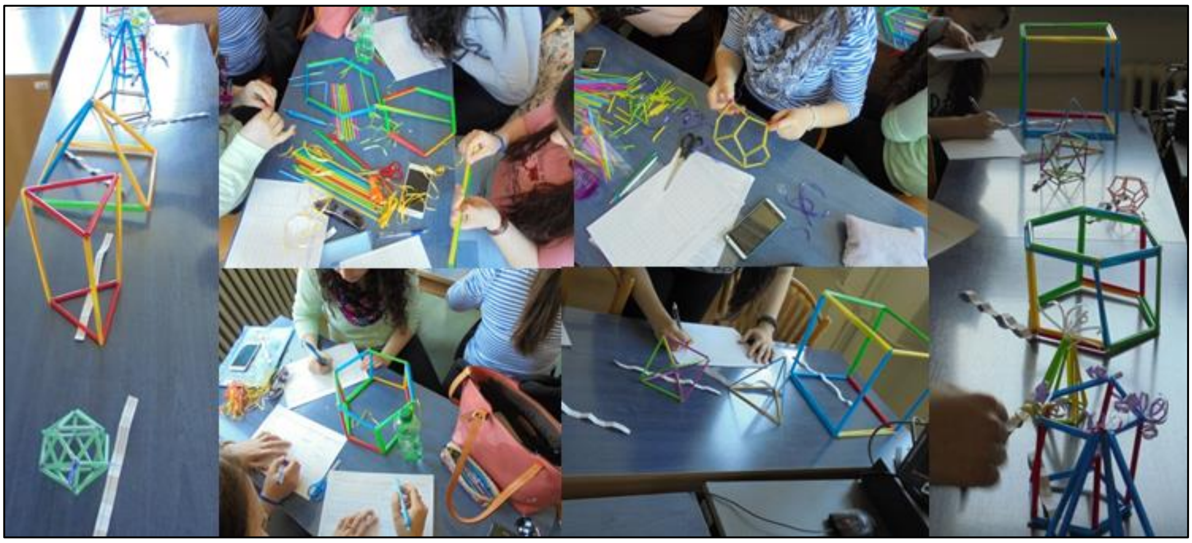

Figure 1

A follow-up task was to fill in a simple table related to the constructed solids. Students were asked to state the names of the solids, the number of their faces, edges and vertices, the shapes of the faces, and the number of the bases. The correct solution is shown in Fig. 2, the light orange shading is applied on those solids which we focus on in this contribution.

	Name of solid figure	Number of			What planar shapes are the faces of this solid?	Does this solid have any bases? If yes, how many?
		faces	edges	vertices		
1.	Cube	6	12	8	Square	No
2.	Regular tetrahedron	4	6	4	Equilateral triangle	No
3.	Regular octahedron	8	12	6	Equilateral triangle	No
4.	Regular dodecahedron	12	30	20	Regular pentagon	No
5.	Right-regular hexagonal prism	8	18	12	Regular hexagon, rectangle/square	Yes, 2
6.	Right-regular hexagonal pyramid	7	12	7	Regular hexagon, isosceles triangle	Yes, 1
7.	Right-regular pentagonal prism	7	15	10	Regular pentagon, rectangle/square	Yes, 2
8.	Right-regular pentagonal pyramid	6	10	6	Regular pentagon, isosceles triangle	Yes, 1
9.	Right square pyramid (right-regular four-sided pyramid)	5	8	5	Square, isosceles triangle	Yes, 1
10.	Right-regular triangular prism	5	9	6	Equilateral triangle, rectangle/square	Yes, 2
11.	Regular icosahedron	20	30	12	Equilateral triangle	No

Figure 2

Results obtained from the tables filled-in by students

When analysing the student tables, we focused on right-regular prisms and pyramids. We aimed to explore the level and quality of knowledge that the involved primary teacher trainees possess about planar and solid figures and their constituents. We also wanted to find out what problems may emerge within this topic.

We considered the student solutions correct also in cases when they did not specify the prisms and pyramids as right-regular or the polygons as regular, or when the shapes of bases were stated in an inappropriate column of the table. The least problematic solid turned out to be cube. Out of 50 teacher trainees 49 managed to fill in all the cube-related cells correctly. The remaining solution was incorrect, for the student stated the name of the solid to be tetrahedron instead of cube.

Many problems occurred with respect to right-regular four-sided pyramid. In total seven students incorrectly stated its name to be right-regular *three*-sided pyramid. Perhaps, when filling in the table, students expected the solids in line 9 and 10 to be of the same type, or they could get puzzled by the model of the solid itself. We had not expected such errors, since prisms and pyramids were explicitly named in the instruction for the task (see Fig. 3, blue shading).

1.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of six congruent square faces.
2.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of four congruent equilateral triangles.
3.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of eight congruent equilateral triangles. Four edges meet at each vertex of this solid.
4.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of twelve congruent regular pentagons. Three edges meet at each vertex.
5.	With the use of drinking straws and string, make a skeleton of a right-regular hexagonal prism.
6.	With the use of drinking straws and string, make a skeleton of a right-regular hexagonal pyramid.
7.	With the use of drinking straws and string, make a skeleton of a right-regular pentagonal prism.
8.	With the use of drinking straws and string, make a skeleton of a right-regular pentagonal pyramid.
9.	With the use of drinking straws and string, make a skeleton of a right-regular four-sided pyramid.
10.	With the use of drinking straws and string, make a skeleton of a right-regular triangular prism.
11.	With the use of drinking straws and string, make a skeleton of a solid figure the surface area of which consists of twenty congruent equilateral triangles. Five edges meet at each vertex.

Figure 3

For each of the solids, except for the cube, there were students who failed to state the correct number of faces; in some cases it was evident that they were not sure if also bases are faces, although they managed to identify the shapes of faces, including the bases.

Altogether 34 teacher trainees failed to identify the faces of the right-regular four-sided pyramid, i. e. isosceles triangles. They incorrectly specified it as equilateral. The remaining students just did not specify the type of triangle.

Problems during the activities

With respect to the way the students approached the task to define the selected geometric terms in their own words, we believed they can easily distinguish between planar and solid figures. In most cases it is obvious they have certain comprehension about the meaning of the terms, yet they cannot express it due to lack of comprehension of other important related notions, such as line, segment, altitude, incidence, intersection point. Having analysed the filled-in tables, we conclude that not all the involved teacher trainees can distinguish between planar and solid figures and use mathematical terms correctly, as evidenced in the following statements of the students: “A face is a segment connecting two points; the segments form a three-dimensional solid and are labelled with lower-case letters.” “An edge is a point where two segments meet; the points form a solid figure and are labelled with capital letters.” “A side is a segment which bounds a drawn solid; it is labelled with lower-case letters with respect to the opposite vertex. A solid is two-dimensional.” “A vertex is the point where two lines/half-lines, or two arcs etc. intersect.” “A side is a line determined by two points in a shape.” Also, one student asked during the seminar if “a face is not the same as a side”.

The most problematic seemed to be the issue of bases. Some students did not count bases among faces. Also, they did not realize the point of talking about bases in a solid, e. g. they did not realize there is no point in talking about bases in a cube. Some of the students stated that a cube has six bases, just because all the six faces are congruent.

Working in groups also brought some complications. In one group students almost did not manage to construct the model of a right-regular five-sided pyramid. One member of the group thought that the base face should be a pentagon; yet, the two remaining students persuaded him that it should be a square. The group realized their mistake later in the seminar and re-made the model.

Although the following issue is not related to prisms and pyramids, it has much to do with notions and thought images in mathematics. Having analysed the filled-in tables, we found out that many of the students cannot distinguish between some planar and solid figures. Namely, some students stated pentahedrons to be faces of dodecahedron, instead of pentagons; similarly, some students referred to regular tetrahedron as equilateral triangle.

Conclusion

We conclude that the involved primary teacher trainees have not fully acquired fundamental geometric terms. They do not distinguish between planar and solid figures, or their constituents. This can cause severe problems in mathematics education of pupils at primary level, who might adopt incorrect register from their teachers, which will require re-education later on.

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