

The Examples of ICT Usage to Forming the Mathematical Language's Precision

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Abstract

The forming of the mathematical language's precision is important one of the aspects of teaching the mathematic. It appears that this specific activity uses the advantages of modern technology to some extent. One can hardly ever find any references to this particular issue in the literature. We made an attempt to present an idea to use the elements of ICT in process of forming the mathematical language's precision for the deeper understanding of the geometrical notions. To do so we will use the program GeoGebra and the program WinGCLC (an implementation of the GLC language). The wrong usage of the verbs specifying the geometrical constructions may lead to fallacy of the mathematical notions. The properly prepared ICT tool may be helpful to the teacher in taking care of the mathematical language's precision.

Keywords: didactic of mathematics, process of teaching and learning mathematics, math's language, new technology, geometry

Classification: C50, R20, G10

Introduction

There are four fundamental aspects of teaching mathematics: forming the mathematical notions, solving the problems, leading the reasoning and forming the mathematical language [11]. As it seems, the most difficult component of this process is acquiring and developing skill of proper usage of the mathematical language. The culture of language itself is based on the language correctness, which needs the proper choice of words. Then the significant problem turns up - how to verify the accuracy of the language in relation to the mathematical notions? Unfortunately, the most of the easily accessible teaching aids (based on the modern technologies) isn't orientated towards forming the mathematical language's precision directly. But the support provided by the modern technologies may perform a role of secondary importance – may help to form the mathematical language. In this article we present the examples of such ICT usage.

The situation of appearing of the proper connections between the above mentioned aspects in student's mind is very important in terms of the didactic of mathematic. This interdependence is most noticeable between the proper understanding the notion (and its properties) and the language describing this notion communicatively.

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One should pay attention to the fact that the mathematical object exists in the virtual world of mathematic and the language is necessary to approximate the virtual object by providing the definition of the notion or the sequence of activities leading to the notion. Therefore one deals with the specific instructions written in the natural language and identified with the determined or selected verb.

Let's analyze the aspect of the relations occurring between this components. Below we present the patterns reflecting the relations between the object, the instruction and the distinguished verb.

The aspect relates to the verification of the level of the mastering the material through the following:

the object <-> the instruction <-> the verb

One can analyze the direction of the relation in two ways.

the object <- the instruction <- the verb

In this case one starts with the verbal form and wants to achieve the proper understanding of the object's notion [13] by using the language-the tool helpful to do so. The mistakes in the language's precision may lead to the situation where the object's notion may even not be reflected.

In the second case the direction is contrary to the noted above.

the object -> the instruction -> the verb

A starting point is the existing mathematical object. One wants to achieve the precision of the language by describing this object. Unfortunately, this way has its disadvantages – the description may be wrong, inaccurate or ambiguous.

Finally the verification in both ways leads to feedback and strengthens the connections between the two the most important components of the process of teaching and learning the mathematic.

The aspect in its description needs the metalanguage. In the case of using the DSG (Dynamic Geometry System) tools the interfaces of used applications may play the role of such metalanguage.

The conditions of observations

This paper describes two examples of ICT usage In developing the correctness of mathematical language. Those examples concern the particular group of the students of the intramural complementary mathematics and ICT master's degree course. The observes participated in the observations as they were teachers.

The student had attended two subjects: "ICT" and "ICT in teaching of mathematic" during the first year of the course, the second subject was more orientated towards the IT usage in the process of teaching and learning mathematics. The student had got to know how to use GeoGebra application as well as WinGCLC long before the observations.

Problem

Let's have a closer look at the example – it's an **elementary constructional problem** – to construct a triangle from three given line segments.

This type of constructional problem is rather disliked by most of the students and even by some teachers as it's assumed that it needs a specific approach to an issue. The problem can be seen at various levels of school education. Nowadays the students use the internet to find help for homework and some solutions of the difficult problems. One can often find some post in discussion boards on the internet written by the users looking for the solutions of specific problems, not only constructional.

One of the users tried to help in solving the aforementioned problem and posted the solution as following:

“Take a segment, may it have an a length. Drive the compass into the end of it and sketch a b radius arc. Sketch a c radius arc in the other end of the segment in a similar way. Anywhere the arcs will cross; there will be the third apex of a triangle.”[†]

Nobody moderates such posts in respect of the mathematical language's precision or even the correctness of the language itself. The solution quoted above is rather an instance of free-styled narration with many poorly chosen verbs than the correct and linguistically precise solution of the mathematical problem. Sadly one can note the insensitivity for such comments; nobody is offended by such language. The student asking for help is simply weak quasi-solutions like the one quoted above will only deepen his ignorance and distort his understanding of mathematics. On the one hand the teachers cannot moderate the comments on the internet, on the other they can teach the students the correct mathematical language. The helpful aid is provided by GeoGebra.

The observed group of students attended “Elementary Geometry” subject during the first year of the course and considered an issue of solving the similar elementary constructional problems. It seemed that the students, teachers-to-be, would do their best to develop precise and correct language. It wasn't confirmed in the observations. Sadly, the members of the observed group's language was freely, imprecise, clustered with loaned colloquial words and some mathematical object's notions. As it seems, imprecise language is the result of specific kind of ignorance and negligence developed on the former stages of education. The students follow the same patterns as their teachers; the distortion of mathematics increases.

It has to be realized that the teachers-to-be have to take care of the language's correctness to break the circle, but it is hard, long-lasting and arduous work. Hence the idea of nonstandard DSG program - GeoGebra usage.

[†] In Polish, the correct choice of the verb depends on the context of its use and form of expression. Sometimes the wrong choice of synonym makes the statement incorrect. The verb permissible in everyday speech is unacceptable in a formal speech. These nuances cannot be fully cast in the text translated into English, in which such a significant difference does not occur.

One can find many examples related to developing the geometric notions or discovering the properties of mathematical objects and relations between them in the literature (describing the GeoGebra [12, 14], WinGCLC [1] and DGS program [3] usage) along with the descriptions of the problems solved with those tools. As it seems, those depictions leave the linguistic issue out, even on the official GeoGebra's site [17].

This paper analyses the different issue. The language is the key to describe the activities, operations or the mathematical objects. The DGS program is only a helpful tool dictated by the problem's nature. The examples place emphasis on the language's correctness and not on the problem's solution. One can find the literature dealing with the meaning of language in the didactic of mathematics [6, 7, 9], but the papers related to didactic ICT usage in this particular issue are hard to find.

The Example I – GeoGebra

GeoGebra [15] is an application free of charge and it's available on multiple platforms. It's typical for DGS and it's very popular so we can pass its characteristic over. Later we will only be referring to the several basic operations.

Coming back to the initiated problem we should analyze the language of the classic constructions using a pencil, straight-line and a compass. It turns out that even the superficial analyze of the syntax shows the significant association of the verbs and the tools listed above.

In concrete terms with a pencil it's possible to:

- **choose** the points of the object or **name** them
- **find** or **determine** the object's intersection

Adding a ruler (as straightline) it's possible to:

- **lead** or **draw** the line through two existing points
- **connect** two distinct points into the line segment
- **lead** a ray from the one initial point through the second
- **construct** a polygon

Adding the compass it's possible to:

- **draw** the circle through one point with centre another point
- **draw** the circle with the distinct centre and the distinct radius
- **draw** the angle from the distinct point
- **transfer the measure** of the line segment

It appears that the list of possible construction is short. Every one of the procedures requires the distinct verb. One can find all of the mentioned above procedures among the many of the procedures in the GeoGebra application and personalize the options by hiding the unwanted ones and using the ones listed below (Fig. 1.).

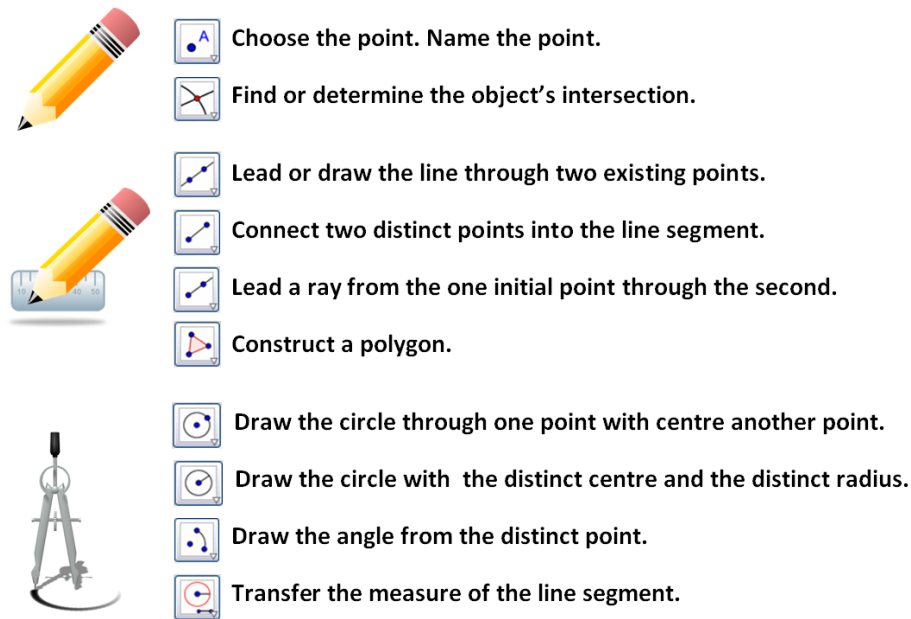


Figure 1

The most important issue of this project is the mathematical object's (shown as "icons" in the program) connection to the proper verbs. This relation in Polish is exact and unambiguous.

This operation leads to forcing the student to use the **specific type of statement during the describing of each the procedure.**

Constructions protocol helps to understand WHAT and IN WHAT ORDER to create to reach the solution. In addition, helps to understand and remember the type of the mathematical objects developed in the process and to form the culture of mathematic.

Obviously, there are no mathematically incorrect instructions in the GeoGebra application, but we can constantly hear and read such colloquial instructions elsewhere, for example the phrase "to sketch" refers rather to an artistic work than to the mathematical construction. The discussion of the questions *–how to solve the problem in GeoGebra? or what kind of icons to use?–* could be helpful to make the students realize the problem and to reveal the mathematical language's incorrectness. It generates the care of the language's precision and the statement's aesthetic. The GeoGebra application clearly states that the line, the ray and the line segment are the very different objects and helps the students to proper classify this objects.

Observations

The observations of the students clearly shown that the exact understanding of the connection:

the object -> the instruction -> the verb

leads to better understanding the notion's properties, to restructure the mathematics, to knowledge's infiltration to different fields of mathematics and beyond. The observed students admitted that it helped them to organize their knowledge.

Another problem is the fact that the complete solution of the constructional problem demands not only the description of the construction itself, but the students often prefer to identify this terms, but this article doesn't analyze that important issue, it needs to be researched and described in future studies.

The Example II – Geometry Constructions Language

Another tool helpful in forming the mathematical language is WinGCLC program-the implementation of Geometry Constructions Language (GCL) [4] on the Windows operational systems with the graphic interface [5]. This specific language was created for constructions of the geometrical objects and for the possibility of describing those constructions on the Euclidean plane. The language has simple and intuitive syntax [10]. The available geometrical constructions are performed with use of the implemented and classic tools-a ruler and a compass.

There are some specific groups of instructions in the language's syntax. In this article we will use only four of them: basic definitions, basic constructions, labeling and printing commands and drawing commands.

The application forces the user to apply the proper syntax of the command, since the user-friendly graphic interface is hard to find in this tool. If the command is incorrect the application will signalize the error; it's the result verifying the correctness. There are three types of such announcements [1]-syntactic errors, semantic errors and deductive errors. The user is determined to apply the proper chosen commands connected with the definition of the constructed object. There is no ease in choosing the keywords; the keywords are the commands-the verbs. It determines the formal notation, but no instrumentally.

The below given example illustrates the notation of the program in the GCL language describing the solution of an elementary constructional problem - let's use the problem quoted before: to construct a triangle from three given line segments (Fig. 2.).

point A 50 40	drawcircle c1
point B 80 40	drawcircle c2
point D 80 40	intersec2 C1 C2 c1 c2
point E 100 40	cmark_b C1
cmark_l A	cmark_t C2
cmark_r B	drawsegment A C2
drawsegment A B	drawsegment B C1
circle c1 A D	drawsegment A C1
circle c2 B E	drawsegment B C2

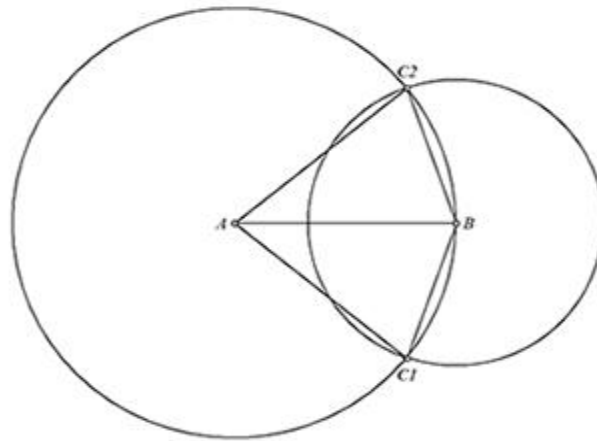


Figure 2

The language's syntax is simple and intuitive enough to assume that the analyze of the aforementioned program won't make an issue and it will be easy to read the idea of connection between the command and the defined object. Therefore we will leave the wider discussion out and refer the curious readers to the program's documentation [16].

It the situation above the verification of the absorbed knowledge leads from the object to the verb. The example will be provided by the following case-on the one of the stages of the construction two distinct auxiliary circles don't have an intersection; the program generates the error result, which is suggesting its essence and the cause in relate to the mathematical object instead of the language's syntax.

Observations

In this program the word means the object's construction. The aspect

the object <- the instruction <- the verb

is highlighted then. In case the program notices some errors or distortions the object isn't responding or it does not exist. The language correctness verification needs the ability of "reading the object".

The students emphasized repeatedly that it seemed WinGCLC language needs the programming abilities rather than mathematical ones, but it appeared that this process is only reverse to the one from the first example. This one demands describing of the constructions protocol with respect of every assumptions and relations between the objects. To construct and to manipulate the protocols one needs to develop the deep understanding of the mathematical objects AND the ability of precise description of these objects. The feedback from the program contains suggestions related to statement's verification and controls the errors. But the proper knowledge is the most important to correct language incorrectness.

Conclusion

The aforementioned examples illustrate the aspect of the verification the absorbed knowledge, but the courses of the examples are opposite.

In the first one (GeoGebra) one can observe the programming of the verification from the distinct verbs to the ideal objects, care of the language's correctness influences the process of forming the object's notion in student's mind very positively.

In the second one (GCL) the verification runs from the constructed object to the description. The errors signalized by the GCL language's processor force the correctness and the precision of the language. But it is noticeable that the correctness is formal in the GCL language made to describe the geometric notion and its properties. Hence it should be stated that the DGS is not only the operation on the program's graphic interface, but it's also an environment using the visual language.

The wrong usage of this tool (without the care of the language's aspect) may lead to an unwelcomed results, like the forming the distorted image of the object or gaining to habit of the distorted formalism.

Not only the solution of the problem, but the annotations also are desired on the every educational stage. The relation between the mathematical objects and the language is natural and strict. The examples of use the ICT to forming the mathematical language's precision show that this relation is not only formed, but also verified.

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