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The Analysis of Students' Solutions of the Word Problem

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Abstract

The paper deals with the analysis of students' solutions of the word problem with real – life context. The solution of task required creating two mathematical models of real-life situation. The first one was the linear equation and the second one an algebraic expression. Individual solutions created the students of 1st grade of four-year program of grammar school. We use a priori analysis to analyse the students´ solution of our tasks. In doing so, we concentrate on the individual phase of the word problem solution. In this way, we wanted to verify not only the use of all parts of the word problem solution by the mentioned students, but also their ability to create mathematical models of the situation. The paper summarizes the results of a priori analysis exemplified by tables and charts.

Keywords: Word problem, a priori analysis, linear equation, algebraic expression.

Classification: D70

Introduction

According to The Mathematics National Curriculum ISCED 2 (The International Standard Classification of Education), the result of teaching mathematics should be the correct use of mathematical symbols, the ability to read continuous text containing numbers, dependencies and relationships with comprehension. The ability to read incoherent texts containing tables, graphs and diagrams. The teaching of mathematics should lead to building the relationship between mathematics and reality, to gaining experience with mathematization of real-life situation and with creating mathematical models. Some students beginning secondary schools have a problem with the solution of word problems with real-life context. They often lack the ability to model real-life situations and the ability to use the language of symbols.

Word problems

Word problems are irreplaceable in the teaching of mathematics. Word problems are helpful tools to practise the gained knowledge. According to Križalkovič (1968) mathematical word problems are problems which express relationship between given and searched numbers by a word formulation. Hejný (2003) defines a word problem as a mathematical problem which requires language comprehension and overlap with life experience. Odvárko (Novotná, 2000) defines a word problem as a task which includes objects, phenomena and situations (with various characteristics and relationships) from the

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different none-mathematics areas. These areas include a common everyday life, various scientific fields and technical practise. Through a word problem students get to a situation where they cannot use any of learnt algorithms immediately. With a task in a word problem students have to find a mathematical definition that fits the given situation the best.

According to Blum and Niss (Šedivý, 2013) word problems should be included into the teaching of mathematics because:

- they are a suitable means for development of general competences of students and attitudes towards mathematics,
- it provides to the students a possibility to be independent in "seeing and judging" and to analyse and understand the use of mathematics,
- it develops both students´ skills in using of mathematical knowledge and also skills in non-mathematical situations,
- it helps students to recognize, understand and keep the mathematical concepts, methods and results.

The word problems solved with the use of linear equations in ISCED 2

Students are meeting the word problems from primary school. At the beginning students solve an easy linear equation with the following methods: trial and error method, table method or intentional object manipulation. Later, in the eighth grade of primary school students meet following concepts: expression, unknown and variable. The tasks are still solved by reflection or illustration. The 9th grade is the grade when the word problems focused on linear equation are solved with the use of equivalent modification.

Methods

We are aiming at the analysis of written answers on word problems of 1st grade grammar school students. At the beginning of the school year the following word problem with real-life context was solved by 23 students:

Water and sewage
Water is the most spread liquid in the world. Before its drinking it is necessary to clean and do disinfection (chlorine). Water supply and draining to the household is provided by *Water Company Clean water* and the company charges the following items for its services:

***Table 1**: The price of water and sewage*

Service	Price Euro/m^3	
	Nett price	**Price with VAT**
water	1,00 €	1,20 €
sewage	0,75 €	0,90 €

Note: Rainfall water fee (the price charged for rainwater drainage from the roofs to public sewerage) is charged according to a roof area in a following way:

- a house with a roof area up to 100 m^2 is charged a flat fee 64,80 Euro per one year
- a house with a roof area over 100 m^2 is charged a flat fee 75,00 Euro per one year

Task 1. Three-room house with a roof area 98 m^2 will consume x m^3 of drinking water per one month. What is a monthly cost of drinking water in three-room house?

Solution: Consumption of drinking water per one month ..... x m^3
Water (consumption x m^3 of drinking water per one month) ..... 1,2x €
Sewage (consumption x m^3 of drinking water per one month) ..... 0,9x €
Rainfall per one month ..... 64,8 : 12 = 5,4 €

$$1{,}2x + 0{,}9x + 5{,}4 = 2{,}1x + 5{,}4$$

Monthly cost of drinking water x m^3 is (2,1x + 5,4) €.

Task 2. Mr. Krasňanský pays to the Water Company for his four-bedroom house with a roof area 101 m^2a deposit fee 30 Euro. How many m^3 of drinking water can Mr. Krasňanský consume without any extra pay per six months?

Solution: Consumption of drinking water per one month ..... x m^3
Water (consumption x m^3 of drinking water per one month) ..... 1,2x €
Sewage (consumption x m^3 of drinking water per one month) ..... 0,9x €
Rainfall per six months ..... 75 : 2 = 37,5 €
Accountable advance per six months ..... 30 . 6 = 180 €

$$1{,}2x + 0{,}9x + 37{,}5 = 180,$$

from here

$$x \doteq 67{,}86 \text{ m}^3.$$

Mr. Krasňanský can consume 67,86 m^3 of drinking water without any extra pay per six month.

The main task consists of the text part followed up with two partial tasks. The partial tasks are aimed creating of mathematical model of the situation in the form of an algebraic expression and an easy linear equation.

Before the task was given to students and their solutions were analyzed, the sample solutions of subtasks had been prepared. The a priori analysis of the expected way of solving the subtasks had been done. The variables of a priori analysis of Task 1 and Task 2 were determined (see Table 2 and Table 3). Then, with respect to the variables, the individual student solutions were analyzed.

***Table 2**: The variables of a priori analysis of Task 1.*

	The variable of a priori analysis of Task 1
P.1	The student solved the Task 1
P.2	The student made a full record of task
P.3	The student created a mathematical model
P.4	The student simplified an algebraic expression
P.5	The student wrote an answer to question

***Table 3**: The variables of a priori analysis of Task 2*

	The variable of a priori analysis of Task 2
P.1	The student solved the Task 2
P.2	The student made a full record of task
P.3	The student created a mathematical model
P.4	The student solved a linear equation correctly
P.5	The student verified the accuracy of solution
P.6	The student wrote an answer to question

Discussion

Figure 1 represents a presence of particular variables in students´ solution of Task 1. We marked the variable "Yes, it is correct" in a following case: the statement substituted by the variable was in the solution and the student used it properly. In case the statement substituted by the variable was used in the solution but improperly, the variable was marked as "Yes, but incorrect". A variable that was not in the solution was marked as "No".

In the Figure 1 we can see that 1st task was solved by 21 students. 21 % of them made a full record of a world problem. Other students only read a task and started to do numerical solution of it. In the analysis we can also see that 7 students did a correct mathematical model of a situation.

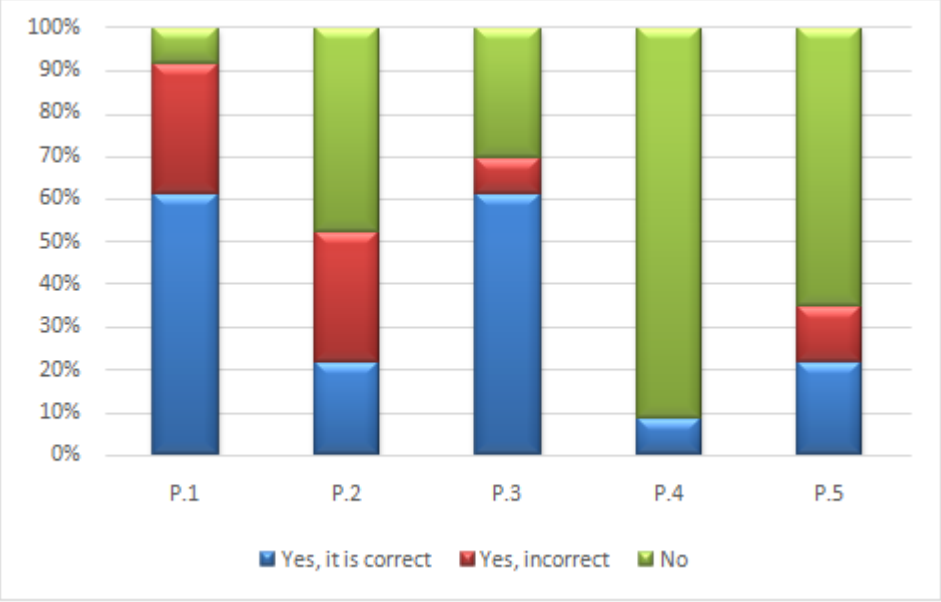

Figure 1

More students were wrong in their precondition that the solution of the task is the price of the rainfall water fee per month. Only two students' solution brought edited algebraic expression representing the price of water and sewage while consumption of potable water is x m^3 per month. The most of students did not write an answer to question because the solution of the task was an algebraic expression.

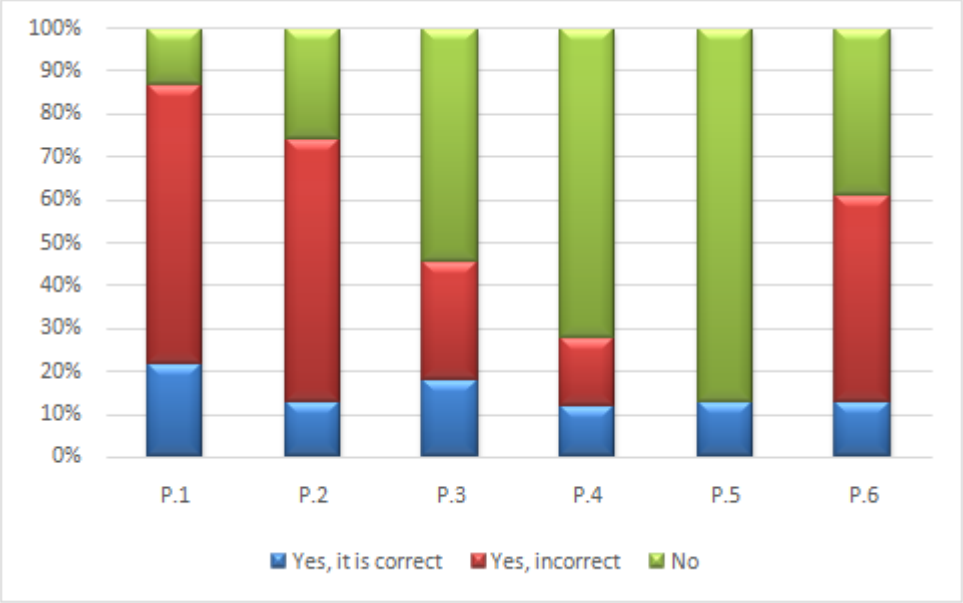

Figure 2

2 students did not solve the Task 2. In 6 of written solutions missed a full record of word problem. Most of students have problem to create a mathematical model of a situation that was an easy linear equation. Wrong mathematical models did not contain rainfall water semi-annual fee, correct sum semi-annual deposit for drinking water. Students, who created correct mathematical model, did not make mistakes in linear algebraic equation solution. 17% of students solved task incorrectly using arithmetical method, 87% of them did not verify their solution and 39% did not write an answer to question.

Conclusion

Based on analysis of students' solutions it can be concluded that students after leaving primary school do not have internalized phases of solving word problem. Many students have problem with creation of mathematical model in the form of linear equation and with algebraic expression. Therefore it is necessary to develop students' skills to create of mathematical models and manipulation with algebraic expression. These skills are necessary for further mathematical learning in higher grades. The reason for the failed task solution could be a lack of understanding assigned tasks inability to obtain necessary information from task or undiscovered appropriate mathematical apparatus.

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