

The Graphic Visualization – the Part of the Solution of Applied Mathematical Tasks

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Abstract

Graphical representation of data and processes in various scientific fields and also in different areas of practical life is useful way for their better understanding. In the paper we concentrated on applied tasks focused on economics, especially in the area of detecting properties of functions. The second area is the usage of the definite integral and its application in the calculation of the consumer surplus and the producer surplus. Via solving of applied tasks using a graphic visualization students gain the necessary knowledge and skills for the study of specialized economic courses. We present the results of the brief knowledge survey which was aimed on mentioned mathematical topics. Graphical sketch of the task solution with application, as the output of solving, help students to create and understand the relation between abstract theoretical term and its practical usage. Collected results confirm that graphic visualization provides opportunities for innovation in the content and methods of teaching mathematics.

Keywords: Mathematics education, functions, definite integral, economics application.

Classification: D20, I20, I50, M40

Introduction

At present an important part of education is the use of quantitative methods and tools of information technology in solving economic tasks, the subsequent application in production and trading companies in the analysis and decision making, in banks, insurance companies, the stock markets and in other areas of production and non-production sectors. In the curricula of students of economical faculties are included subjects like mathematics, statistics and computer science, which provide the necessary basis for solving problems with applications.

Information technologies (IT) are changing the way of acquisition, processing and handling of knowledge and also call for changes in the usage of information. This situation is also reflected in the educational process. The IT phenomenon is very important when working with information and educational aspects of this phenomenon were summarized by Hrmo - Krelová [1] as follows:

- Visualization, which facilitates the imagination of the phenomenon and shortens the learning process,

- Process simulation, which may under different input values to create a model of the behavior of the real process,

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- Interaction between the computer and the user, which is one of the important characteristics of multimedia,

- Interpretation of the curriculum, especially in presentations of inaccessible objects and phenomena.

To the mentioned facts we can add that via methods, forms and means of informatics education can be promoted self-discipline of students, their autonomy and creativity. IT tools offer teachers opportunities to motivate students to improve their learning outcomes and promoting study activities generally. If the mathematical concept is explained and also visually represented, then the proper understanding and ability to operate with him in solving practical examples is easier. Therefore, graphs and visualization of solutions in applied problems is an important part of teaching mathematics with application. It is also necessary to provide the study subject with current study literature and use it during seminars [2], [3].

Many studies have dealt with the relationship of students to the tools of information technologies and their ability to use them. The relationship of students to new technologies was investigated by Mišútová - Mišút [4] and their research has confirmed the validity of the hypothesis: there is a positive correlation between attitudes to new technology in the mathematics teaching and performance in tests involving tasks aimed at the basic factors of creativity.

Students of the Slovak University of Agriculture in Nitra come from all kinds of secondary schools. At the Faculty of Economics and Management the majority of students come from grammar schools and business academies. They are expected to study terms and methods of higher mathematics with their applications in the other subjects of specialization. However, levels of their mathematical knowledge from secondary schools are different. In this paper we focused on applied tasks with graphic visualization and students abilities to use them in the solution of mathematical tasks with economic application. We present selected tasks with application in which we can use graphical visualization. The results of realized survey give us information about students' knowledge and skills to use graphs of functions in solving tasks.

Types of applied tasks with graphic visualisation

Economics uses not only basic elementary mathematical operations, but also knowledge of higher mathematics. Because mathematics is not one of the most popular subjects among students and, therefore, teachers strive to make it more attractive for students by means of: - Implementing of proper, especially applied problems and tasks into the curriculum,

Proper presentation of the subject via multimedia and Internet which have a global positive

impact [5].

Students learn to calculate mathematics in specific mathematical situations tasks purely of mathematical content, which lacks real context. The result is that the student controls theory and can compute tasks [6]. One of the main tasks of education should be the increase in student's motivation to learn. Based on the obtained results it can be concluded that learning objectives are not clear to students, who probably assume that the study subject is not needed for their further education and professional application [7].

The principal topics in compulsory mathematics courses at the Faculty of Economics and Management (FEM) are functions. Plotting graphs of functions belongs to the content of the

first lessons. In the tasks with applications by functions will be represented: costs, revenues, expenses, earnings, consumption and so on. The compulsory mathematical subjects (in the winter and summer semester) include the following topics with graphs of functions:

- An overview of elementary functions properties and graphs,
- Asymptotes of the graph of a function,
- Detection properties and graphs of functions monotonicity, concavity.
- Calculation of definite integral,
- Calculation of the area of a plane figure.

Samples of application tasks

1 Functions and applications in economic analysis

Basic functions of economic analysis are often expressed through elementary functions. As an example, we give the total cost function CN(x), which can be formally written in the form of a polynomial of the third degree.

Thus, the expression for a function is:

$$CN(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3,$$

$$CN(x) = N_f + N_V,$$

 $N_V = A_1 x + A_2 x^2 + A_3 x^3$ - is a variable component of the total cost,

 $N_f = A_0$ - is fixed component of the total cost.

In Figure 1 and Figure 2 there are graphs of polynomial functions displayed by software GraphSight v.2.0.1. [8].





Figure 1: $f: y = 0.3x^3 - 0.4x^2 - 0.3x + 1.2$

Figure 2: $g: y = -x^3 + 0.4x^2 + 2x + 1.2$

2 Definite integral and its applications - the consumer surplus and producer surplus

A consumer surplus (*CS*) is defined as the amount of money that the consumer is willing to pay more for a given product than the current market price. Producer surplus (*PS*) is an economic measure of the difference between the amount that the producer of a given goods receives and the minimal amount that the producer would be willing to accept for the

goods. The producer always tries to increase his producer surplus by trying to sell more and more at higher prices.

In the scheme of market mechanism (see Figure 3) there are used these main concepts: D(x) is a function of demand, S(x) is the supply function, the equilibrium point is $E = [\bar{x}, \bar{p}]$ where \bar{x} is the equilibrium quantity and \bar{p} is the equilibrium price that satisfy relations $\bar{p} = S(\bar{x}), \bar{p} = D(\bar{x})$.

Then formula for consumer surplus is $CS = \int_{0}^{x} [D(x) - \overline{p}] dx$.

Formula for producer surplus is $PS = \int_{0}^{\overline{x}} [\overline{p} - S(x)] dx$.



Figure 3: Scheme of market mechanism

Knowledge survey

Higher education of future economists and managers includes the subject Microeconomics, in which the knowledge about the market mechanism is used. We decided to find out the level of the skills and knowledge the first-year students from the main concepts in this area.

During academic year 2014/2015 we performed a survey, which was focused on solving problems with graphical interpretation. The tasks in the test were solved by students of study program Business Economics (number of respondents 60) and students of study program Economics and Management of Agro-food Sector (number of respondents 20), studying in 1st year at SAU Nitra in full-time study. Thus, the number of tested students together was 80. The number of tasks in the test was 4 and scoring of each task was from 0 to 6 points. For visual comparison of the results, we divided the students' score results into intervals < 0, 2.5 >, < 3, 4.5 >, < 5, 6 >.

So we have created a test with the following tasks:

- To find and formulate the properties of the function from its graph (monotonicity, concavity, boundedness),
- To sketch the model of the market mechanism: demand supply,
- To interpret the found results: equilibrium quantity, equilibrium price,
- To calculate consumer and producer surplus via definite integral.

Results and discussion

Graphical representation of the survey results by the intervals is in the Figure 4. The 1st and the 2nd tasks are related - examine students' knowledge about graphs of functions and their properties. Demand and supply function were given by formula for linear and quadratic function. From the graph in Figure 4 we see that more than half of respondents obtained less than half of the points for these tasks. 3rd task is in relation with 1st and 2nd task, too. The equilibrium point and its coordinates are calculated from the equation S(x) = D(x). The results indicate that more than half of respondents had difficulties in solving this task. We can state that the best results students obtained in 4th task. Students should use the above mentioned formulas for the surpluses and calculate the definite integral. A lot of numerical errors and missing steps in calculating integrals influenced their point score.



Figure 4: Results of knowledge survey

Conclusion

The decision making in the planning, production and sale is conditioned by sufficient information and suitable methods used for solving economic problems. Mathematics provides proper methods and means for solving application tasks and for finding the correct answer. In many tasks students are asked to sketch graph of the function that expresses the dependence between the variables that represent given data. Visualization of the solutions of the tasks, illustrating by graphs, tables and images will have positive impact on the efficiency of mathematical education.

Students perceive the theoretical aspects and practical application of knowledge separately without inner relations. This finding was confirmed by the survey results. Therefore, it is necessary to explain to students mathematical terms not only in verbal form but also through graphic illustrations. Tasks should combine theoretical knowledge with the applications in order to recognize relation between abstract theoretical term and its practical usage.

Graphic illustration and visualization of the solutions provides opportunities for innovation in the content and methods of teaching mathematics:

- The range of fundamental topics and applications should be adapted to the study course,
- To present mathematical methods in applied tasks,
- The solution of the tasks connect with graphic illustrations and images,
- To add graphical interpretation of the solutions in the assignment of seminary projects.

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