

# **Testing of the Fundamental Mathematical Knowledge**

Zuzana Malacká<sup>\*</sup>

Department of mathematics, Faculty of humanities University of Zilina, Univerzitná 8215/1, SK - 010 26 Žilina

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#### Abstract

The study results of mathematics at a university are determined by knowledge of secondary curriculum. The content and the scope of teaching of mathematics are reducing at secondary schools and the graduation exam in mathematics is not compulsory. In the paper we present on understanding of fundamental mathematical knowledge in the first study year of bachelor study at the Faculty of Civil Engineering University of Žilina in Žilina . Also there are listed the most common deficiencies and suggestions to improve the situation.

Keywords: Mathematical knowledge, pedagogical experiment, research, study results, mathematical thinking .

Classification: B10, B40

#### Introduction

Mathematics is an activity concerned with logical thinking, spotting patterns, posing premises and investigating their implications and consequences. It also involves the study of the properties of numbers and shapes, the relationship between numbers, inductive and deductive thinking and the formulation of generalizations. Mathematics is a creation of the human mind and therefore becomes primarily a way of thinking thus facilitating problem solving.

Mathematics is very important object in modern society, so mathematics education must to satisfy the needs of society. In fact, mathematics education is always behind social development. In this system, our children and students, the new masters of their own learning, are asked to somehow discover the ways of arithmetic by trying to figure out worded math problems. Today math isn't only about numbers, it's about words and theories, as if the curriculum was written by folks, who hate the clear logic of pure mathematics. Mathematics education must never stop reforming. It can become a power which pushes society forward [2].

Slovakia is a developed country. Mathematics education is taken into account by government and education departments. They support and help mathematics education reform. Where is mathematics education reform from? From technology? From textbooks? Or from the mathematics education system?

<sup>\*</sup>Corresponding author: <u>zuzana.malacka@fpv.uniza.sk</u>

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## **Research setting**

I have taught mathematics on the university level for 19 years. Students have been cheated by not having been taught the basic fundamental skills that are essential to an understanding of the subject. Memorizing multiplication tables and mastering long divisions are as fundamental to math as learning the alphabet is reading and writing. Many of our students are ill prepared for higher level mathematics. Students need to memorize basic mathematics facts in order to focus on higher- level concepts. As a teacher I know that give some examples of what you believe is wrong with the curriculum. This curriculum taught without an experienced teacher mitigating its effects by insisting on mastery of fundamentals results in the majority of students becoming frustrated and losing their math confidence entirely.

For measuring the high school mathematics knowledge of the students I made the first year students to write a test, according to that we can say that in the previous knowledge of many our students there are big differences. For the purpose of experiment I randomly chose the group 85 students from the 1-st grade of Faculty of Civil Engineering in Žilina.

Test

**1.** Which of the following numbers are natural, integer, rational or irrational numbers? -2;  $-\frac{3}{2}$ ;  $\frac{30}{15}$ ;  $\frac{4}{12}$ ; 5,07; 9; -81,5; 4, $\overline{1}$ ;  $\pi = 3,14159...; \sqrt{3}$ ;  $-\sqrt{7}$ ;  $\sqrt[3]{8}$ ;  $\sqrt[5]{3}$ ;  $\sqrt[3]{-64}$ . **Answers:** correct **27** partially correct **32** incorrect **26** 

## Analyzing of some solutions of students:

Some students correctly identified numbers but they could not designate sets of numbers. The biggest problem for many students was to determine rational and irrational numbers.

2. Edit the expressions

a) 
$$(\sqrt{3} - 2x)(\sqrt{3} + 2x)$$

b) 
$$\sqrt{\frac{9}{4}x^2}$$

c)  $\sqrt[3]{(1-x)^3}$ Answers: correct **33** 

partially correct **31** 

incorrect 21

**Analyzing of some solutions of students:** Some students didn't know the formula

a) 
$$(\sqrt{3} - 2x)(\sqrt{3} + 2x) = 3 - 2\sqrt{3}x + 4x$$

b) 
$$\sqrt{\frac{9}{4}x^2} = \frac{3}{2}\sqrt{x^2}$$
  
c)  $\sqrt[3]{(1-x)^3} = (1-x)^2$ 

**3.** Edit the complex fractions

a) 
$$\frac{2-\frac{x^2+y^2}{xy}}{\frac{x}{y^2}-\frac{2}{y}+\frac{1}{x}}$$
 b)  $\left(\frac{u+v}{u-v}+\frac{u-v}{u+v}\right):\left(\frac{u+v}{u-v}-\frac{u-v}{u+v}\right)$  c)  $\left(\frac{2ab+a}{a+2bc}\right):\left(1-\frac{c}{a}\right)$ 

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Answers: correct 19
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partially correct 24

incorrect 52

#### Analyzing of some solutions of students:

The students made wrong steps in operations with fractions.

They made frequently mistakes when they shortcut the fractions and they often forget to write a conditions.

a) 
$$\frac{2 - \frac{x^2 + y^2}{xy}}{\frac{x}{y^2} - \frac{2}{y} + \frac{1}{x}} = \frac{2 - \frac{x^2 + y^2}{xy}}{\frac{xyx - 2y^2x + y^2y}{y^2yx}} = \frac{2 - x^2 + y^2}{x - 2x + y} = \frac{2 - x^2 + y^2}{-x + y}$$
  
b) 
$$\left(\frac{u + v}{u - v} + \frac{u - v}{u + v}\right) : \left(\frac{u + v}{u - v} - \frac{u - v}{u + v}\right) = \left(\frac{u + v + u - v}{u - v . u + v}\right) : \left(\frac{u + v - u - v}{u - v . u + v}\right) = \left(\frac{2u}{u - v . u + v}\right) : \left(\frac{0}{u - v . u + v}\right) = 0$$
  
c) 
$$\left(\frac{2ab + a}{u - v}\right) \left(\frac{2b + 1}{u - v} + \frac{2b + 1}{u - v}\right) = \left(\frac{2b + 1}{u - v . u + v}\right) = \left(\frac{2b + 1}{u - v . u + v}\right) = 0$$

c) 
$$\left(\frac{2a+2b}{a+2bc}\right): \left(1-\frac{a}{a}\right) = \left(\frac{2b+1}{1+2bc}\right): \left(\frac{a}{a}\right) = \left(\frac{2b+1}{1+2bc}\right) \cdot \left(\frac{a}{1-c}\right) = \frac{2b+1a}{1+2bc\cdot 1-c}$$

4. Solve equations using the decomposition

a) 
$$2x^2 - 8 = 0$$

b)  $x^2 + 3 = 0$ 

c) 
$$x^2 - 5x + 4 = 0$$

Answers: correct17partially correct40incorrect28

Analyzing of some solutions of students:

a)  

$$2x^{2} = 8$$

$$x^{2} = 4$$

$$x = 2$$
b)  

$$x^{2} + 3 = 0$$

$$x^{2} + 3 = (x + \sqrt{3})^{2}$$

$$x = -\sqrt{3}$$
c)  

$$x^{2} - 5x + 4 = 0$$

$$x^{2} - 5x + 4 = 0$$
  

$$x(x - 5) + 4 = 0$$
  

$$x(x - 5) = -4$$
  

$$x = 2, x = 3$$

5. In a class, the ratio of boys to girls is  $\frac{4}{5}$ . If there are 12 boys in the class, how many girls are there?

Answers: correct29partially correct0incorrect56Analyzing of some solutions of students:

The students solved this example right or they had the problem to express the text mathematically and they didn't solve it.

- 6. Which numbers are Pythagorean triples? Write yes or no.
  - a) 3, 4, 5 b) 4, 5, 6 c) 24, 45, 51 Answers: correct **36** partially correct **0** incorrect **49**

Analyzing of some solutions of students:

			$c^2 = a^2 + b^2$
a)	3, 4, 5	yes	$E^2 - 2^2 + 4^2$
			$3^{-} = 3^{-} + 4^{-}$ $25^{-} = 9^{-} + 16^{-}$
			25 = 25
b)	4, 5, 6	no	
			$6^2 = 4^2 + 5^2$
			36 = 16 + 25
			36 ≠ 41
c)	24, 45, 51	yes	

 $51^{2} = 24^{2} + 45^{2}$ 2601 = 576 + 20252601 = 2601

Some students could not use Pythagoras' Theorem. Even many of them did not know it. They only guessed or used triangle inequality.

7.	Draw the graphs of functions				
	c)	24, 45, 51	yes	24 + 45 > 51	
	b)	4, 5, 6	yes	4 + 5 > 6	
	a)	3, 4, 5	yes	3 + 4 > 5	

a) y = 2x - 3 b) y = 7x

c) y = -3x + 5 d) y = -4x

Answers: correct25partially correct48incorrect12

### Analyzing of some solutions of students:

Students substituted several values for x and they found values of y as a solution of a given equation. Then they locate the points on a coordinate plane. They had a correct solution. Others only guessed.

8. Draw the graphs of functions

Answers: correct	34	partially correct 32	incorrect	19	
c) $y = x^2 + 4x + 4$		d) $y = -x^2$	d) $y = -x^2$		
a) $y = x^2 - 4$		b) $y = -x^2 - 1$			

Analyzing of some solutions of students:

Students substituted several values for x and they found values of y as a solution of a given equation again. Then they locate the points on a coordinate plane. They had a correct solution because these students knew that the graph of the quadratic function is a parabola. Others only guessed again.

- 9. Convert to the indicated units
- a)  $324,58 mm = \cdots cm$
- b) 98,7  $m = \cdots dm$
- c)  $4 \ cm^2 = \cdots mm^2$
- d) 2.3657  $dm = \cdots mm$

Answers: correct13partially correct33incorrect39

## Analyzing of some solutions of students:

Some students had problems with converting of units.

- a) 324,58 mm = 3245,8cm
- b) 98,7 m = 9,87dm
- c)  $4 \ cm^2 = 40 \ mm^2$
- d) 2.3657 *dm* = 2365,7*mm*

### The most common mistakes:

- Students have a problem to modify the complex fractions
- They make numerical errors
- Students have problems mathematically express the conditions
- They have problem to solve more difficult examples, which require several mathematics procedures
- Students don't know read the text carefully
- Students do not know the formula well and often unable promptly select the right one needed to calculate
- They have little developed spatial imagination
- They cann't draw enough visual image
- Very often they make numerical errors
- They have problems with converting of units
- They do not specify the conditions under which this term has meaning
- They have problems to adjust polynomials (e.g. the removal of braces, as well as setting aside factor-1 before brackets)
- They make wrong steps in operations with fractions

- They make frequently mistakes when they shortcut the fractions. Students without embarrassment shortcut only one addend in the sum (e.g.  $\frac{2a^2+13b}{7a^2} = \frac{2+13b}{7}$  etc.)
- They perform the wrong operations with the opposite polynomial
- They very often make the numerical errors in the substitution of a negative number
- They use dificult procedures (e.g.  $\frac{2z+1}{z^2-1} + \frac{2+3z}{z-1} = \frac{...}{(z^2-1)(z-1)}$ )
- Students are not used to verify the result obtained
- The problem is often to write a term  $\frac{t^2}{a} l^2$  as one fraction
- They do not use the correct formula (e.g.  $(t-9l)^2 = (t-9l).(t+9l)$  or  $(a+b)^2 = a^2 + b^2$ )
- Very often are leave out the brackets (for example x + 1.x 1 instead of (x + 1).(x 1))
- The biggest problem for students is the verbal text to express mathematicaly
- Students not thinking about the credibility of the result and they often take for granted completely nonsensical result
- Students often lack a creative approach to solving the problems

Meaning of all these errors is to provide teachers what they should be examine. Identified deficiencies should lead to a search of causes and their gradual removal.

## Conclusion

Mathematical knowledge is fundamental to the understanding and development of science and technology as well as being applicable to many areas in the social sciences. We see in the last few years the rapid decreasing of the mathematical knowledge and in general the education level of students in higher education. My experience is the same. It is also alarming that the skill to apply mathematics and the fundamental mathematical knowledge of graduated engineers show a decline [3].

One of the keys to enhanced progress in mathematics for young people is a sound mathematical foundation at primary level, delivered with enthusiasm, confidence and flair. The most important element in mathematics education is teacher [4]. The role of the teacher and school is crucial in people progress as the best teachers are able to motivate, educate and improve understanding for children from a range of backgrounds. The teacher controls the course of teaching and learning, guides and helps the students' understanding and sets the class goals. The best teachers are able to motivate, educate and improve understanding for children a range of backgrounds. But teachers are often concerned with is student's success in exams, not student's understanding of and use of mathematical concepts [1]. If teacher develops student who think mathematically, his class will be really enjoyable for both him and the student. Mathematical thinking is an important goal of schooling. It is important as a way of learning mathematics and for teaching mathematics too. Mathematical thinking is a highly complex activity. Let us strive to teach for understanding of mathematical concepts and procedures, the "why" something works, and not only the "how". The relationship between the "how" and the "why" - or between procedures and concepts - is complex. One doesn't always come totally before the other, and it also varies from child to child. And, conceptual and procedural understanding actually help each other: conceptual knowledge (understanding the "why") is important for the development of procedural fluency, while fluent procedural knowledge supports the development of further understanding and learning. Teacher should often test a student's understanding of a topic by asking him to produce an example, preferably with a picture or other illustration.

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