

Rediscovered Anaglyph in Program GeoGebra

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Abstract

In this paper we study several applications of geometry beginning with visualization of spatial object in the plane up to modelling of special curves and surfaces in computer graphics. We show the usefulness of the dynamic program GeoGebra for better visualization of geometric relationships in space using anaglyphs.

Keywords: Anaglyph, dynamic geometric program, GeoGebra, Monge projection, curves and surfaces in CAGD.

Classification: 97G80

Introduction

Many people say that they are not very good in space vision or that they have poor spatial sense. The typical belief is that the child is either born with spatial sense or not. But we know that rich experiences with shape and spatial relationships (consistently provided) can develop the spatial sense to a large extent.

Dynamic geometric program GeoGebra [1] is a very useful helpmate for rigorous visualization of geometric relations in plane and in space. For visualization the shapes in the space we can use two types of outputs. The first one is the widely used parallel projection of the given 3D object (can be changed to central projection, but in this case the parallel lines project into nonparallel lines and the picture could be not suitable for teaching basic geometric properties or relationships). The second and the newest possibility for visualization of spatial objects in GeoGebra is to choose anaglyph. The program creates two central projection images into one picture in red and cyan colour. Using anaglyph filter glasses we can see the spatial object in space partly in front of and partly behind the screen. This is a perfect tool for training spatial geometry abilities, mainly for students with lower spatial sense.

Anaglyph

Anaglyphic stereogram (anaglyph) is the name given to the spatial effect achieved with encoding each eye's image using chromatically opposite (usually red and cyan, occasionally red and green in Europe) filters. "Anaglyph" is derived from two Greek words meaning "again" and "sculpture". (So we again find ourselves discovering this technique in the 21st century.)

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Anaglyph images contain two differently filtered coloured images, one for each eye. The images are created from two central projection images projected from the centres approximately 6.5 centimetres apart, which is the centre distance typically between human eyes. The central projection of the object projected from the left centre is green/cyan coloured, the image projected from the right centre is red coloured. Viewing an anaglyph stereo image is then easy: we need a red filter over the left eye and a green/cyan filter over the right eye. The eye that is covered with the red lens will see only the green/cyan image in black. Similarly, the eye that is covered with the green/cyan lens will see only the red image in black. If colours on the glasses fit with the colours on the printed or displayed material, we must not see the red image through the red glass with left eye, respective the green image through the green glass with right eye. Each eye will see only the image prepared for it (see Figure 1) and so the perception of depth will be created.



Figure 1

The first method to produce anaglyph images was developed in 1852 by Wilhelm Rollman in Leipzig, Germany. In 1858, a French gentleman named Joseph D'Almeida used this technique to project anaglyphic stereo lantern slides onto a theatre screen [2]. The audience viewing the exhibition was adorned with red and green goggles to witness the very first 3D slide show ever.

William Friese-Green created the first 3D anaglyphic motion pictures in 1889, using a camera with two lenses, which were first shown to the public in 1893. Anaglyphic films called "plastigrams" enjoyed great popularity in the 1920s. These used a single film with a green image emulsion on one side of the film and a red image emulsion on the other. In 1922, an interactive plastigram, entitled "Movies of the Future," opened at the Rivoli Theater in New York [2]. Anaglyphic images have been used in comic books, newspapers, and magazine ads. In 1953, 3D comic books were invented and distributed with red and green "space goggles." But anaglyphs were used not only for entertainment. Some descriptive geometry textbooks with anaglyph illustration also arose. One of them is the textbook [3]; its first edition is from the year 1959.

Anaglyphs are used today in science for instance to map the topography and geology of the planets and moons. Stereo images can reveal geologic features of the planet not otherwise visible. Just as the early stereogram images allowed people around the world to experience

different places and events, stereo images of the planets help us to experience these different worlds in a more tangible way. Scientists at the Jet Propulsion Laboratory used anaglyphic images to look at the surface of Mars using pictures sent back by the Pathfinder spacecraft. This allowed the scientists to experience the planet in a more familiar way. The features of the surface and the rocks could be analysed in more detail using these images. The Viking spacecraft that visited Mars in the 1970s also returned some images in stereo, as did the Apollo missions to the Moon in the 1960s (Figure 3) and 1970s [2]. Figure 2 shows the image presented by NASA, a two-colour anaglyph from the Mars Pathfinder mission (red-cyan glasses are recommended to view the images correctly).



Figure 2



Figure 3

Using anaglyph in dynamic program GeoGebra

Monge projection is a short name for orthogonal projections to two perpendicular (horizontal and vertical) planes.

The double-view orthogonal projection is named after Gaspard Monge, the French mathematician, the inventor of descriptive geometry. Monge projection has a wide range of uses in technical drawing and computer graphics. This is the reason, why Monge projection forms a basic part of the subject Constructive Geometry in teacher training study. Despite the fact that the program GeoGebra does not support Monge projection, it is possible to

display a front view and a top view of the objects quite successfully. As an example we show the solution of an intersection problem from the university textbook [4]. The task requires determining the intersection of two triangles *ABC* and *MNP* in Monge projection. The coordinates of the vertices are given as *A*[-1, 0, 6], *B*[-4.5, 4.5, 0], *C*[2, 7, 1.5], *M*[0.5, 1, 0.5], *N*[3.5, 3, 4.5], *P*[-2.5, 7, 5].



The *x* axis of the plane coordinate system in the graphics screen of GeoGebra is used as an x_{12} axis in Monge-projection. That means that the top view of the triangle *ABC* is given by (see Figure 4) A_1 [-1, 0], B_1 [-4.5, -4.5], C_1 [2, -7], and the front view is given by A_2 [-1, 6], B_2 [-4.5, 0], C_2 [2, 1.5].

The top view and the front view of the triangle MNP are given similarly. In general, the top view of a space point X[a, b, c] is a plane point $X_1[a, -b]$, and the front view is given by $X_2[a, c]$. The further solution of the task is identical with the solution in paper form. The advantage of this dynamical geometry form of solution is that the coordinates of basic points are variable. We can create a new task from the previous one and confront the solutions. This variability has only an assumption that the front view and the top view of each point move on a perpendicular line to the x₁₂ axis.

A problem appears, when some students are not able to imagine these objects in 3D space.

Program GeoGebra can be helpful in this case too. We can use the 3D version and display the objects in space (see left part of the figure 5).

To increase the perception of depth and to have full stereo image, we can use the newest function of the program, which divides the 3D picture into two central projections in green and red colours, i.e. creates anaglyph of the original picture. The right side of the figure 5 shows the anaglyph of the left side picture (red-cyan glasses are recommended to view the image correctly). After using the rotation function of the program and view the image through red-cyan glasses, we receive a full spatial experience with rotating black-and-white objects in front of and behind the screen.





The second ideal usage of anaglyphs in geometry teaching appears in the subject Curves and Surfaces [5] for computer graphics study. Anaglyph is a great support for students with lower level of space vision. As an example we show the bilinear surface with parabola diagonals. Students are able to compute the equation of the curve on the surface but often, they are not able to imagine it in space. Figure 6 shows the situation in parallel projection, and figure 7 shows the same situation in space (of course, red-cyan glasses are recommended to view the image correctly). In the pictures, the control points of the diagonal curves appear. To increase the spatial experience, we can choose a rotation function of the program.



Figure 6





Conclusion

The aim of the paper was to demonstrate the usefulness of anaglyph in the program GeoGebra for teaching spatial geometry relationships. At first we dealt with the history of anaglyph, than we discussed, how a perfect 3D experience was created. We have showed the usefulness of this feature of the program GeoGebra on the seminars of geometry in subjects Monge projection and Curves and Surfaces.

Acknowledgement

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