Project "E-matik+"
Education for Mathematics Teachers

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Abstract

From the point of view of effective education, improving the competences of mathematics teachers is a necessary process. The form of their education is just as important as the content of their further education. On the Faculty of Mathematics, Physics and Informatics, Comenius University, we are providing accredited continual education for teachers of mathematics in e-learning form for several years, from the area of geometry and didactics of mathematics. Through this distance method, their digital competencies are being improved, as well. This contribution is describing the continual education from its content and formal perspective, as well as experience regarding educating teachers.

Keywords: Teacher education, digital technology, e-learning.

Classification: D40, U50, U70

Introduction

Digital technologies (DT) offer teachers a possibility to make use of new educational methods, e.g. the constructivist approach, controlled search, workshop method or peer instruction method. DT are very suitable for project teaching, too. Teachers can make use of blended learning, flipped classroom method, etc.

Last but not least, the computers are used for electronic testing when knowledge of the pupils is measured. Generally accepted theoretical basis in this area is Technological Pedagogical Content Knowledge (TPACK) as a framework for the integration of technology within teaching.

According to (Mishra, P. & Koehler, M. J., 2006), see Figure 1, teachers must understand how technology, pedagogy, and content interrelate, and create a form of knowledge that goes beyond the three separate knowledge bases. Teaching with technology requires a flexible framework that explains how rapidly-changing, protean technologies may be effectively integrated with a range of pedagogical approaches and content areas (Mishra, P. & Koehler, M. J., 2008).
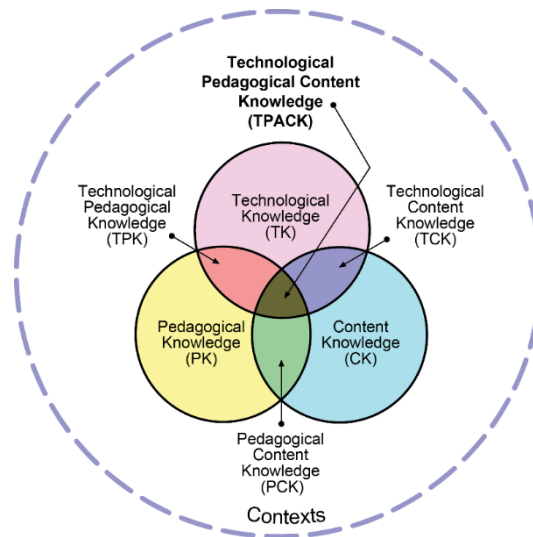
Figure 1 *(Technological Pedagogical Content Knowledge: A new framework for teacher knowledge, 2006)*

New technologies are driving necessary and inevitable change throughout the educational landscape. Effective technology use, however, is difficult, because technology introduces a new set of variables to the already complicated task of lesson planning and teaching. The TPACK framework describes how effective teaching with technology is possible by pointing out the free and open interplay between technology, pedagogy, and content. Applying TPACK to the task of teaching with technology requires a context-bound understanding of technology, where technologies may be chosen and repurposed to fit the very specific pedagogical and content-related needs of diverse educational contexts (Mishra, P., Koehler, M. J., & Kereluik, K., 2009).
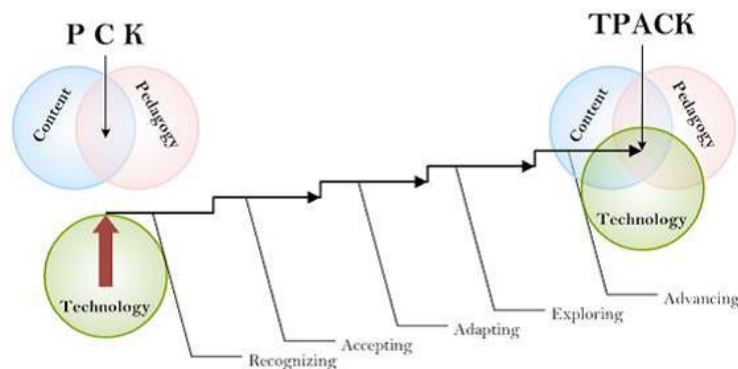

Figure 2 Mathematics teacher TPACK standards and development model. (Image from http://www.citejournal.org/vol9/iss1/mathematics/article1.cfm)

The questions of TPACK and mathematics teaching were in detail dealt with by Niess et al. Figure 2 shows the visual description of teacher levels as their thinking and understanding merge toward the interconnected and integrated manner identified by TPACK.

Besides benefits of DT for teaching, the computers are used for electronic testing when knowledge of the pupils is measured.

During mathematics classes, pupils can make use of digital technology in various ways:

- during numerical calculations so they can concentrate on the solution of the problem itself;
- for visualisation, modelling and simulation of problems and thus to obtain such a graphical representation of the problem, which pushes them towards a solution;
- as a source of educational materials e.g. e-books or videos, interactive educational materials;
- drilling exercises, a pupil can make use of electronic working sheets or e-tests to evaluate himself.

From a didactic point of view, we can use these resources in different teaching methods:

- method of controlled discovering,
- project education (this is another rising form of mathematics teaching, which is gaining new possibilities in the digital environment. Students use mostly cooperative learning, which is enabled by m-learning),
- peer instruction method (in this case, the tablet is used as a voting device),
- "The flipped classroom" method, etc.

From an educational point of view, teaching with a tablet is appropriate for both individual and collective work.

According to (Lukáč, S., 2001) the use of a computer while teaching can contribute to fulfilling some didactic functions:

- motivation - the computer can contribute to forming positive attitude towards learning
- informative function
- driving function - highly interactive educational programs can control the exchange of opinions between the subject of education and the computer
- rationalization function - proper integration of the computer into the educational process can support the differentiation of procedures and methods of teaching regarding the relations towards individual students
- control function - using the computer can make diagnosis and evaluation of educational results much more effective
- communication function - computer aided teaching supports mutual communication between students
- social function – work in groups.

At the present, it is important for teachers of mathematics in Slovak schools, that they improve their digital competencies, expand their knowledge from selected domains in mathematics and get acquainted with new educational methods and forms. From the digital technologies point of view, the following tools are suitable for teaching mathematics: GeoGebra, HotPotatoes, software for interactive whiteboards, e.g. OpenSankore, which is open source, etc. Familiarization with these tools is the content of some modules of continual education. For the distance form of education we chose LMS Moodle. Even the communication via LMS Moodle is improving their digital competencies.

Project "EMATIK+"

At Comenius University, we have dedicated already 10 years for preparing future teachers of mathematics and the topic of using ICT in teaching mathematics on elementary and secondary schools. For this purpose, two courses are available. In the first course, students are acquainted with the wide range of software (and other ICT means), which are available on elementary and secondary schools in Slovakia. In the second course, they are acquainted with the didactics of teaching mathematics using ICT. There are several dozens of diploma works with this topic. Through continuing education of teachers, during the years 2006 – 2009, with the help of the ESF EU „EMATIK" grant, we trained more than 1000 teachers in the area of using ICT in education via e-learning. Based on a good experience with e-learning form of education, in 2012 we accredited continuing education as a course "Digital technologies in teaching mathematics on elementary and secondary schools". At the present, within the project KEGA, we are testing new forms and methods of this continuing education. The success of the EU depends on the skills of workers and the creative economy for development of information society. Continual vocational training has by CEDEFOP European Centre double meaning: it supports economic growth and social cohesion. The project "E-matik +, lifelong training of teachers of mathematics" (KEGA 094UK-4/2013) solved by two departments at Comenius University, is to create a Moodle e-learning portal, including selected video-tutorials, adapted according to feedback from current practice in three recently accredited lifelong education programs for teachers of mathematics in Slovakia:

1. Digital technologies in mathematics at primary and secondary schools
2. Geometry and computer graphics in the further education of teachers of mathematics and its use in primary and secondary schools (GaPG)
3. Creating teaching and testing in digital form

Short title of the project is created after keywords Mathematics, ICT, e-learning. The plus sign represents the main added value – improving professional education. We plan to optimize the cost of software tools (open information technology, global movement FOSS), travel costs of emerging social network of teachers, following the successful ESF project EMATIK (finished 2008). Our long-term goal is to prepare the international cooperation and follow-up by for English or German language areas. Educational materials will be created, verified and tested in practice with a unique target group – teachers of mathematics who directly co-create the quality of education our children and youth.

The necessity to improve professional competences of mathematics teachers (both in digital technologies and new didactic approaches) is a result of practice needs as well as the goals in the new state education program. We plan to create a digital content highly adapted to the recent progress, easy to access and very modern.

The goals of continuing education are focused on perfecting the professional competences in the field of digital technologies, necessary for standard pedagogic work (within teaching mathematics on elementary and secondary schools). The goal is to develop and innovate the competences of teachers regarding digital technologies. The educational program is a summary of independent modules. GeoGebra plays a key role in two of them:

Module 1: Freely available software in teaching mathematics and methodics of its use in teaching mathematics on secondary schools (GeoGebra, HotPotatoes, Open Sankore, etc.)

Teachers learn how to properly embed software into the educational process, to assess the suitability of the form of the used software and they gain an overview about the accessible software and its placement into education

Module 3: Dynamic mathematics and methodology of its use in teaching mathematics on secondary schools (GeoGebra, Cabri geometry, etc.)

Teachers learn how to use the GeoGebra software and Cabri geometry in teaching mathematics on secondary schools. They learn how to properly embed digital technologies into the educational process and they will be able to apply the software according to the latest forms of teaching mathematics and to effectively use the given software in a digital classroom.

Figure 3 *The continuing education portal, available at http://elearn.ematik.fmph.uniba.sk.*

The project "E-matik +" aims to create and use Moodle-based e-learning portal. The education itself is planned for one semester and it is concluded with final presentation and a discussion on the topic of the completed course in front of a three-member examination commission.

We present a few examples of tasks, presented in the courses.

Example 1

A significant part of the module „Projective geometry" of the education program GaPG is dedicated to finite structures – affine and projective planes. We try to explain some basic notions and properties at the pupils of primary and secondary schools level. Our goal is to find some connection(s) between affine and projective planes and the real life. As an example of useful application is as follows:

In a firma a computer net with local subnets is needed with some requirements (for optimalizing and security):

- every two computers must be connected using exactly one local subnet,
- for every two subnets there is exactly one computer being connected in both subnets,
- there exist at least four computers of which no three are in the same local subnet.

How much computers must our firma buy in order to build this net if they need at least 15 and can afford less than 35 computers? Without knowing that it is a model of finite

projective plane one can just guess and increasing the number of computers makes this problem more difficult.

The similar, more general and more discussed is the „communication problem", where a set of users would like to communicate with each other (e.g. is participants of a telephone system). After having formulated the requirements for a communication system, we translate this into geometric language. This leads to notions of „linear space" and – as a special case – the „finite projective plane" which correspond (depending on the requirements) to those communication systems. (See e.g.: Albrecht Beutelspacher And Ute Rosenbaum: Projective Geometry: From Foundations to Applications. Cambridge University Press 1998, ISBN 052148277 1 hardback, ISBN 0 521 48364 6 paperback, available at http://www.maths.ed.ac.uk/~aar/papers/beutel.pdf).

Secondary school teacher Kinga Horvath – participant of the above mentioned e-learning course – had a final presentation on finite affine planes with this example (Horvath, K., 2001):

There were 16 racers registered to a motorcycle race in which only 4 can use the competition path. Organize groups of 4 racers in such way that every two racers will fight in competition exactly one times.

In geometric language – if the racers are „points" of finite (affine) plane then the starts (groups) of the competition are „lines" of this plane.

After discussing how to solve this real-life problem one can get this table of starts:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
0	4	8	12	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
1	5	9	13	4	5	6	7	5	4	7	6	6	7	4	5	7	6	5	4
2	6	10	14	8	9	10	11	10	11	8	9	11	10	9	8	9	8	11	10
3	7	11	15	12	13	14	15	15	14	13	12	13	12	15	14	14	15	12	13

This can be (as a model of a finite affine plane) illustrated by a picture (Figure 4)
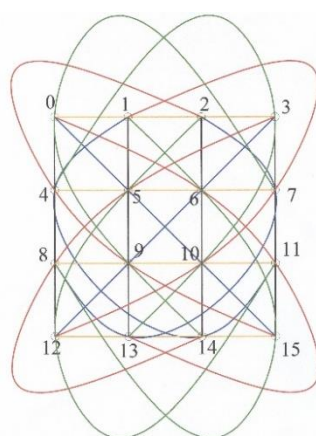

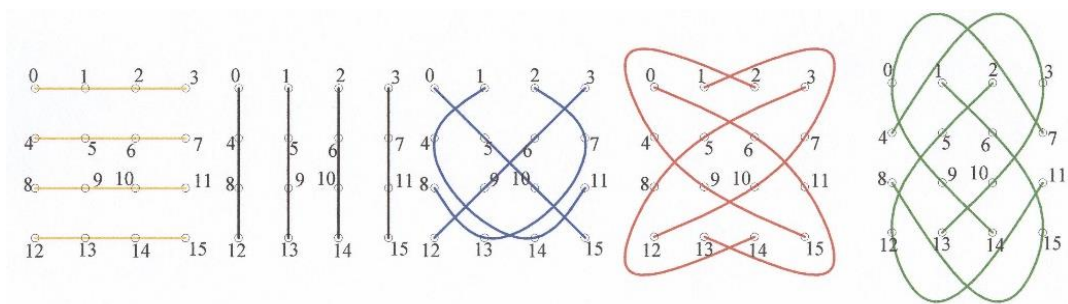*Figure 4* Modelling the solution by Kinga Horvath

Figure 5 Modelling the solution by Kinga Horvath, part 2.

Example 2

During the course Digital Technologies in Mathematics at Primary and Secondary Schools, teachers get acquainted with useful Java applets, which they can use during the explanation of educational materials on an interactive whiteboards or as an individual task for pupils working with a computer (see Figure 6).
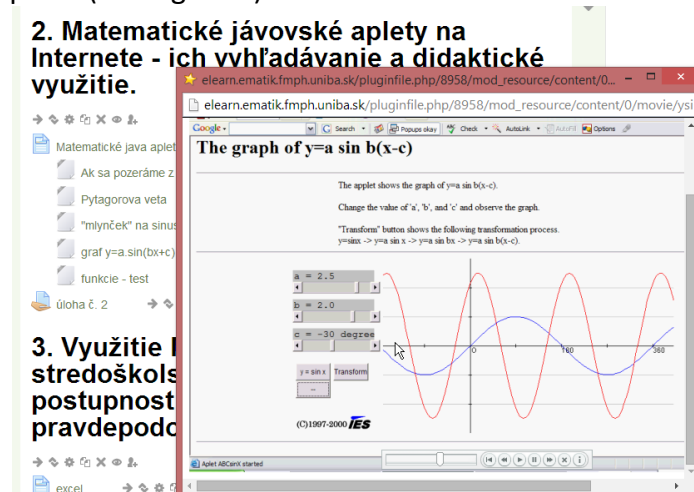

Figure 6 A screenshot of an interactive Java applet activated.

In this course, teachers of mathematics also get acquainted with the open source software GeoGebra, which has a very wide range of use in school mathematics. They get an overview on how to create GeoGebra files and applets. In addition, the learn how to use this software on concrete problems in different didactic situations and in different educational methods and forms. We are presenting an example:

We would like a big yard for our pet!

Siblings Eva and John would like to build a fence for their pet's yard. Their parents allowed them to build a fence around an area in the shape of a rectangle next to the stable, but they have only 15 meters of netting. What should be the size of the paled area if they want to have it as large as possible for their pet to play on? See picture. If we choose x as one side of the rectangle, then we get the function of the area y depending on the side x.

$$y = x \cdot (15 - 2x)$$

How will the values of x and y look if we can measure only with a precision of 1m? 0.5m? 0.1m? Let's make a table. Where will be the maximum for the area?

The solution using GeoGebra is presented in Figure 7. This example can be used as a motivational task on an interactive whiteboard, during which the teacher uses the controlled discussion method and a solved GeoGebra worksheet. A different use of this very example is, if the teacher uses the guided discovery method and students work in pairs next to a computer. Here students solve the problem by themselves in GeoGebra and discover some soundness – the function, its graph and maximum.
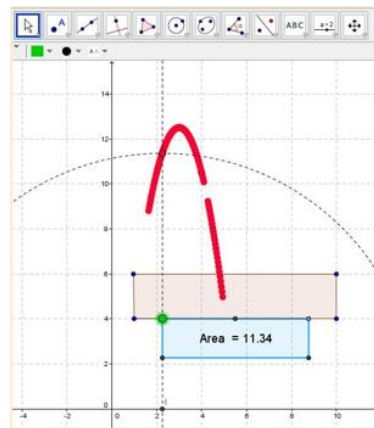

***Figure 7** The size of area maximum found!*

A popular software for teachers is also the free software HotPotatoes. Using this software, teachers can create electronic tests. E-tests are not only considered as a tool for measuring the knowledge of the students, but mainly as electronic worksheets, which enable the students to advance individually while solving problems and with immediate feedback (Figure 8).
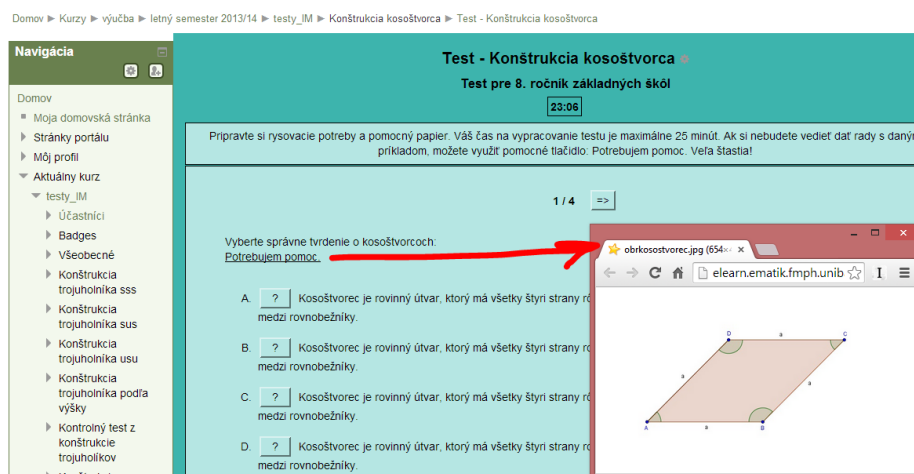

Figure 8 A sample use of HotPotatoes.

Conclusion

Experience from the past years has shown, that for teachers and future teachers of mathematics, the content we provide in our course is very useful and so is its e-learning form. In the State educational program (Mathematics, ISCED 3A), it is recommended to use other forms of education, such as in a computer classroom. Pupils should learn to use ICT for searching, processing, saving and presenting information, that should make difficult

calculations and procedures easier for them. The correct and effective use of digital technologies is undoubtedly beneficial for teaching mathematics. Because of these facts, it seems this continuing education of teachers of mathematics for raising their professional competences to use digital technologies, is reasonable.

Marc Prensky defined four degrees of introducing digital technologies into education – typically a four-step process: 1. Dabbling, 2. Doing old things in old ways, 3. Doing old things in new ways, 4. Doing new things in new ways (Prensky, 2005).

At the majority of Slovak schools, the situation is somewhere at degree 1 or 2. An effective introduction of digital technologies, however, requires at least a level 3, which means that teachers must learn new educational methods and forms and possibly even to innovate the content. Our courses are oriented precisely on improving the competencies of teachers, so that they can get to level 3 or 4, which will improve the effectiveness of teaching mathematics.

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