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Analysis of Errors in Student Solutions of Context-Based Mathematical Tasks

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Abstract

The submitted contribution is concerned with analysis of errors made by students when solving context-based mathematical tasks. The contribution comprises evaluation of four tasks which were designed by the authors of the contribution within project KEGA 015 UKF – 4/2012 in Slovakia. Altogether 56 first and second year students of primary teacher training university master programme were asked to solve the tasks. The errors in the student solutions were identified and classified primarily following Newman´s error categories and additional categories suggested by the authors of the contribution, who furthermore propose 13 error subtypes. In total 127 inappropriate solutions of the four tasks were included in the evaluation. The authors present a sign scheme and a correspondence map of student errors based on statistical analysis. As evidenced by the analysis, students make similar errors when solving tasks of the same type. The objective of the authors is to identify accurately and classify the error types occurring in student solutions.

Keywords: Context-based mathematical tasks, student errors, KEGA 3700109.

Classification: 97D70

Introduction

The importance of relating mathematics education to everyday life is commonly known and generally accepted. In labour market employers are often disappointed by the graduates' inability to use mathematical knowledge in practice, and thus they demand that more emphasis be put on practice-oriented mathematics education (Graumann, 2011). The main objective of this movement is to develop students' ability to apply mathematics in everyday life, which is seen as the core goal of mathematics education (Biembengut, 2007).

The use of mathematics in everyday life has also been among the main concerns of researchers within project KEGA 015 UKF – 4/2012 (Fulier et al., 2014) at Constantine the Philosopher University in Nitra, Slovakia. The research project, as well as the Programme for International Student Assessment (PISA) organized by the Organisation for Economic Co-operation and Development (OECD), has been interested in applications of mathematics in daily life. In PISA tasks there are used such real word problems which require quantitative reasoning, spatial reasoning or problem solving (OECD, 2003). The Slovakian KEGA project researchers also based their priorities on the above mentioned ideas. Therefore the main focus of the project was on designing appropriate learning material – new context-based tasks for pupils at elementary schools covering primary and lower secondary education

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levels, inspired by PISA tasks. Despite the importance of contexts for learning mathematics, several studies (Wijaya, Heuvel-Panhuizen, Doorman and Robitzsch, 2014) indicate that contexts can also be problematic for students when they are used in mathematics tasks. In addition, if students are expected to be able to use mathematics in practical life situations, teachers have to be well-trained in this area of mathematics teaching.

Within KEGA project the research was conducted among hundreds of elementary school pupils in Slovakia, and the concern of measurements was their ability to solve context-based mathematical tasks. Although the results were evaluated through common statistical methods (Csáky – Cafiková, 2015), no similar analysis of errors was performed. We believe that the presented analysis can provide us with additional useful information about the difficulties that pupils and students face when solving mathematical tasks of this kind.

The concern of the submitted contribution is to identify the errors made by teacher trainees in their solutions of contextual mathematical tasks.

Theoretical background

There are many ways to define context in mathematics education. Generally, it can be stated that „contexts are recognized as important levers for mathematics learning because they offer various opportunities for students to learn mathematics“ (Wijaya et al., 2014). PISA study defines context as a specific setting within a ´situation´ which includes all detailed elements used to formulate the problem (OECD, 2003). The ´situation´ refers to the part of the students´ world in which the tasks are placed. The term context-based mathematics task refers to a task situated in real-world setting which provides elements or information that need to be organized and modelled mathematically (Wijaya et al., 2014).

Analyzing student errors in solving context-based mathematical tasks

The analysis of student errors in solving context-based mathematical tasks can be done with the use of Newman's model, the Newman Error Analysis (1977, 1983). This model uses five categories of errors based on the process of solving mathematical word problems: errors of reading, comprehension, transformation, process skills, and encoding. In the submitted contribution this model is compared with other models, such as stages of Blum and Leiss´ modelling process (Maass, 2010) and the PISA mathematization stages (OECD, 2003), which were designed for solving context-based mathematics tasks. After detailed comparison, Wijaya et al. (2014) considers Newman's model to be suitable for analysis of student errors in solving context-based mathematical tasks (Wijaya et al., 2014).

Research question

Within the Slovakian KEGA project several hundreds of context-based mathematical tasks were designed for elementary school pupils, and the tasks were also tried and tested in elementary school environment. The project results indicate that the designed tasks are appropriate for the purpose of improving mathematical literacy of elementary school pupils.

The submitted contribution aims to investigate results of teacher trainees for primary education level who took a test designed for pupils of the last grade in lower secondary education level, identify and evaluate the errors the teacher trainees made when solving the context-based tasks. The presented statistical approach can be further used for the final evaluation of test results obtained within KEGA project.

The research question is as follows:

What types of errors do teacher trainees for primary education level make when solving context-based mathematics tasks?

Method

Mathematical tasks

The test used for the research was administered to collect data about student errors when solving context-based mathematical tasks. The test was designed on the basis of PISA tasks and KEGA project tasks. The test consisted of five tasks. The tasks covered several mathematical issues. In order to solve the tasks common mathematical knowledge from elementary school curriculum was sufficient. Although the test consisted of five tasks, four of them were covered in the research evaluation. The one task, focused on elementary statistics and probability, was not covered in the evaluation since it was impossible to identify accurately types of student errors. The first tasks, titled *Smartphone* required elementary knowledge of geometry and measure. The second task, titled *SMS*, covered elementary curriculum about numbers, variables and computation. The third tasks, titled *Electricity*, tested students' elementary knowledge of relations, functions, tables and diagrams. The fourth evaluated task, titled *Cars*, was focused on elementary logic, reasoning and proofs in mathematics. Students were allowed to solve the tasks for 35 minutes in total, similarly to KEGA project within which elementary school pupils had from 7 to 8 minutes for a task.

Participants

Altogether 56 first and second years students of primary teacher training university master programme took the test. After finishing their studies, the students will teach also mathematics in primary education level. During the spring term in 2015 they attended an optional subject at Department of Mathematics, Faculty of Natural Sciences, Constantine the Philosopher University in Nitra, within which they were ask to solve the context-based mathematical tasks. The students were not informed about the test in advance, since the objective was to examine their ability to use their mathematical knowledge when solving context-based tasks without any special preparation.

Procedure of coding the errors

To investigate the errors, only the students´ incorrect responses, i.e., the responses with no credit or partial credit, were coded. Missing responses, which were also categorized as no credit, were not coded and were excluded from the analysis because a student error cannot be identified from a blank response. For the analysis of student errors the model proposed by Wijaya et al. (2014) was used, based on Newman´s error categories and in agreement with Blum and Leiss´ modelling process and PISA´s mathematization stages (see Table 1).

Table 1

Error type	Sub-type	Explanation
Comprehension	A – Misunderstanding a keyword	Student misunderstood a keyword, which was usually a mathematical term.
	B – Error in selecting information	Student was unable to distinguish between relevant and irrelevant information.
Transformation	C – Procedural tendency	Student tended to use directly a mathematical procedure without analyzing whether or not it was needed.
	D – Taking too much account of the context	Student´s answer only referred to the context/real world situation without taking the perspective of the mathematics.
	E – Wrong mathematical operation/concept	Students used mathematical procedure/concepts which are not relevant to the tasks.
	F – Treating a graph as a picture	Student interprets and focused on the shape of the graph, instead of on the properties of the graph.
Mathematical Processing	G – Algebraic error	Error in solving algebraic expression or function.
	H – Arithmetical error	Error in calculation.
	I – Error in mathematical interpretation of graph	Student mistakenly focused on a single point rather than on an interval.
	J – Improper use of scale	Student could not select and use the scale of a map properly.
	K –Measurement error	Student could not convert between standard units or from non-standard units to standard.
	L – Unfinished answer	Student used a correct formula or procedure, but they did not finish it.
Encoding	M	Student was unable to correctly interpret and validate the mathematical solution in terms of the real world problem.

Statistical analyses

The objective of the analyses was to identify types of errors in relation to specific mathematical tasks. The aim was to find out what error types are typical for which tasks in the test. Data were evaluated with the use of $\chi^2 -$ test of independence, correspondence analysis and sign scheme.

Two nominal variables were used in the statistical analyses – the first variable assuming m categories (m error types) and the second variable assuming n categories (n evaluated tasks). Each task was assigned a score (number of students who made an error of specific type in the task), so that the contingency table $m \times n$ would be formed.

In total, we obtained 224 responses (number of tasks solved by all students in total), which included 82 correct responses (36%), 127 incorrect responses (i.e. no credit or partial credit, 57%), and 15 skipped tasks (7%). The error analysis was carried out for the 127 incorrect responses. The analysis revealed that 57% of the 127 errors were mathematical processing errors, and 25% were transformation errors. Encoding errors (11%) and comprehension errors (7%) were less frequent.

For a detailed list of identified errors according to error sub-types see Table 2.

Table 2

Sub-types	Smartphone	SMS	Electricity	Cars	Active Margin
A	1	3	2	0	6
B	1	2	0	0	3
C	6	0	0	1	7
D	0	3	0	0	3
E	0	6	0	1	7
F	15	0	0	0	15
G	13	2	0	0	15
H	2	0	0	2	4
I	1	0	27	1	29
J	0	0	17	0	17
K	0	0	0	0	0
L	3	4	0	0	7
M	0	0	1	13	14
Active Margin	42	20	47	18	127

The standard tool for testing dependence in contingency table is $\chi^2 -$ test. However, its result does not reveal any specific information about the structure of the variables. Correspondence analysis (CA) is a descriptive and survey method, it does not cover any tools for testing the statistical significance of the obtained models. In quantitative research, CA can be used as a part of any stage of the categorical variables processing – from preparation to the presentation of results. (Hebák et al., 2007, p. 169). CA serves to reduce multidimensional space of line and column profiles (see Řezanková, 2007) in the table to a space of two dimensions, if possible, so that relations between categories of the quantities classified in the contingency table can be shown as legibly as possible – in a planar graph. The closer the points are to each other in the correspondence map, the higher the similarity between the corresponding categories. Groups of similar categories can be also identified regarding the position of the points with respect to the main axes (Spálová, Fichnová, Szabová, 2013).

Sign scheme is another way of graphical representation showing the relations in the contingency table. It is based on testing the fit between the observed and the expected frequencies. The sign scheme is formed on basis of the normalized residuals (Hendl, 2006). The greater their absolute values, the more intense the relation between the categories of the variables. Normalized residuals are transformed to mathematical signs (Řehák – Řeháková, 1978). Signs $+$ mean that the pair of the two categories (the error type and the task) evaluated in the cell of the contingency table is over-represented, signs $-$ mean that the pair of the two categories evaluated in the cell is under-represented (Szabo, 2015).

Results

The tested null hypothesis H_0 was that the type of an error does not differ with respect to the type of the solved task. The significance level of the hypothesis testing was $\alpha = 0{,}05$. The statistical testing was processed with the use of SPSS software. The results of the statistical processing were as follows: the value of the test statistic $\chi^2 = 272{,}75; p = 0{,}000$. The tested null hypothesis is thus rejected at significance level $\alpha < 0{,}01$. It means that the type of an error depends on the type of the task. Accordingly it makes sense to do a correspondence analysis for the data. The first dimension contributes to the overall inertion

(2,148) by 40,7 % of the total inertion, the second dimension by 32,7 % of overall inertion that together sum to 73,4 % of the overall inertion, so they together contain sufficient information about the correspondence of row and column categories. The correspondence map (Figure 1) and the sign scheme (Table 2) serve to illustrate this dependence.

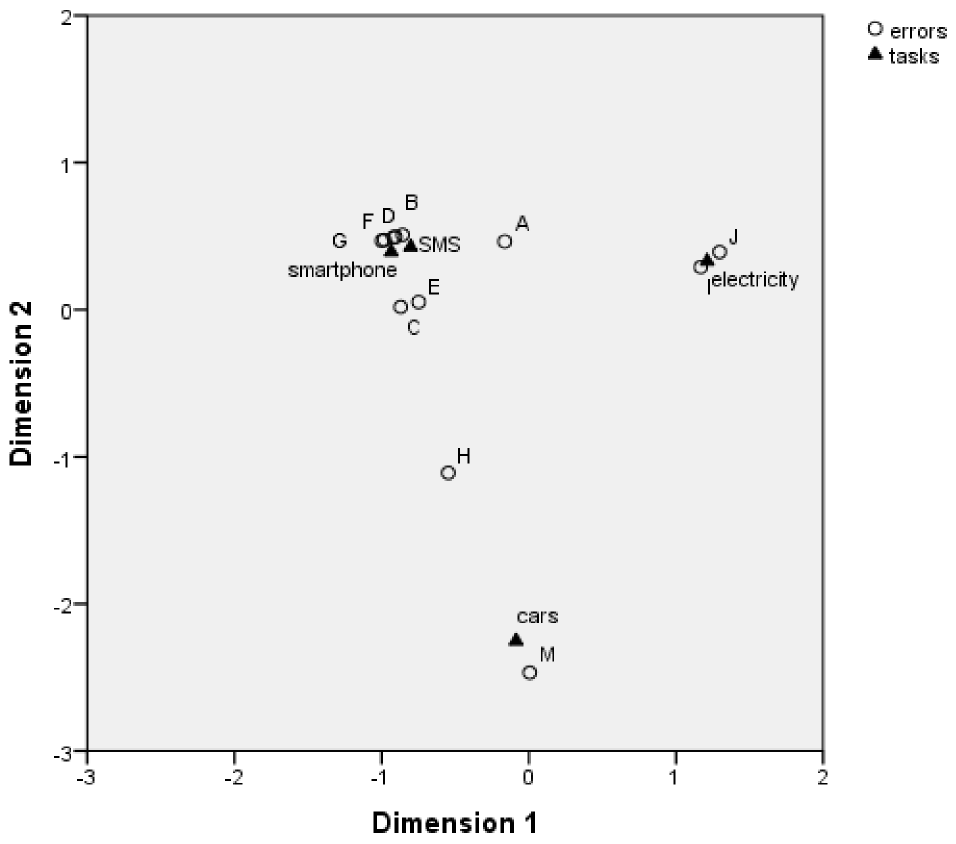

Figure 1: *Correspondence map*

Table 2: *Sign scheme*

	Smartphone	SMS	Electricity	Cars
A	0	+	0	0
B	0	+	0	0
C	++	0	-	0
D	0	+++	0	0
E	0	+++	-	0
F	+++	0	--	0
G	+++	0	--	0
H	0	0	0	+
I	---	--	+++	0
J	--	0	+++	0
L	0	++	-	0
M	--	0	-	+++

The statistical results show that when solving the first task, which was focused on curriculum covering geometry and measure, students significantly often made errors of F, G and C type. Students drew unsuitable pictures, used inappropriate method for computation, or did not use Pythagorean Theorem. The second most frequent error made when solving this task was using an inappropriate formula without any drawings and without any mathematical considerations, which led to an incorrect result.

In the task focused on curriculum about numbers, variables and computation, students were asked to determine the most advantageous monthly fee programme offered by the phone call service provider. The most frequent errors were of comprehension and transformation type: A, B, a D, E. For students it seemed difficult to identify crucial information and set appropriate mathematical conception in order to determine the most favourable monthly fee programme. Many students answered the question without any mathematical considerations. The second most frequent error (type L) was leaving the task unfinished after having performed correct computation, in other words students forgot to answer the question asked in the task wording.

In the third task, which was focused on relations, functions, tables and diagrams, students made the most errors, and the errors were of type I and J. Students were expected to gain information from the graph illustrating electricity consumption. Students were not able to comprehend the scaling in the graph, and thus failed to determine the intervals which would provide them with the answer to the question.

In the fourth task, which was focused on elementary logic, reasoning and proofs in mathematics, the least errors were recorded. The recorded errors were of type M. Students were not able to interpret their results. Students were expected to work with data about car theft rate in Slovakia during recent years. Although many students managed to compare the numbers correctly, they failed to interpret their results. Some students made minor arithmetic mistakes when trying to express the data in percents (error type H).

Conclusion and Discussion

In the presented research the analyses focused on students´ errors when solving context-based mathematics tasks. The objective was to determine if primary teacher trainees are able to use mathematics in daily life situations. The analyses show that when solving tasks of specific type students make similar errors. The types of errors occurring in specific task types have been determined and categorized. As the error analysis shows, the most errors were made by students when solving tasks examining their ability to interpret graphs and diagrams and in geometrical tasks, namely the errors were of type F, G, and I, J, which – apart from error G (algebraic error) – can be reduced by solving context-based mathematics tasks within school instruction more frequently. The authors of the submitted contribution believe this to be of utmost importance, since the ability to gain information from graphs and diagrams and to become conscious of crucial details is getting more and more needed in real life situations.

References

Biembengut, M. S. (2007). Modelling and applications in primary education. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 451-456). New Yok: Springer.

Fulier, J., Vrábelová, M., Vrábel, P., Kmeťová, M., Rybansky, Ľ., Nagyová-Lehocká, Z., Kóšová, M., Uhrinová, E., Lednický, L., Csáky, A. (2014). *Zvyšovanie matematických kompetencií žiakov nižšieho sekundárneho vzdelávania (ISCED 2). Všeobecné otázky a výsledky pedagogického experimentu.* Nitra: Univerzita Konštantína Filozofa v Nitre.

Graumann, G. (2011). Mathematics for problem in the everyday world. In J. Maasz & J. O´Donghue (Eds.), *Real-world problems for secondary school mathematics students: case studies* (pp. 113-122). Rotterdam: Sense Publishers.

Hebák, P. et al. (2007): *Vícerozměrné statistické metody (3)*. Praha: Informatorium.

Maass, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik*. 31(2), 285-311.

Newman, M. A. (1977). An analysis of sixth-grade pupils´ errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin.* 39, 31-43.

Newman, M. A. (1983). *Strategies for diagnosis and remediation.* Sydney: Harcourt Brace Jovanovich.

OECD. (2003). *The PISA 2003 assessment framework – Mathematics, reading, science, and problem solving knowledge and skills.* Paris: OECD

Řehák, J. ,Řeháková, B. (1978). Analýza kontingenčních tabulek: rozlišení dvou základních typů úloh a znaménkové schéma. *Sociologický Časopis*. 14(6).619-631.

Spálová, L., Fichnová, K., Szabová, E. (2013). Some Alternatives in Statistical Analysis of Semantic Differential. *GrKG/Humankybernetik*. 54(4), 190-196.

Szabo, P. (2015). Závislosť medzi mierou pozornosti venovanej predvolebnej kampani a volenou stranou v parlamentných voľbách. *Forum Statisticum Slovacum.* 11(1), 98-103.

Wijaya, A., van den Heuvel-Panhuizer, M., Doorman, M., Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students´ errors. *The Mathematics Enthusiast.*11(3), 555-584.