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Some Famous Problems of Discrete Geometrie

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Abstract

Newton number, Kepler's conjecture, Happy End Problem and some packing and structural problems will be discussed. Highlighted are open problems.

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Classification: 05B40, 52C35

1 Introduction

For the problems of discrete geometry is symptomatic their clearness. The formulation of the problem is very simply and the understanding obviously requires no special knowledge. However, the solution often requires certain dose of ingeniousness and the method of the solution changes from the problem to the another problem. Obviously, my choice of the problems is subjective and many articles and books have been written about most of them. Many of them can be found in the references and the substantial part of those is easily attainable. For this reason I will present here only brief information about selected problems.

2 Newton number

Newton number (also kissing number) $N(B^d)$ in $d-$dimensional space is defined as the maximum number of non-overlapping unit spheres that can be arranged such that they each touch another given unit sphere. Trivially, $N(B^1)=2$ and easily can be proved that $N(B^2)=6$. In the dimension 3 a famous discussion between Isaac Newton and David Gregory took place on the campus of Cambridge University in 1694. The right result $N(B^3)=12$ was correctly proved by Schütte and van der Waerden [ScvW53] only in 1953 and a little later very elegant proof was published by Leech [Le56]. Extremely symmetric lattices found Leech [Le64], Musin [Mu03] proved $N(B^4)=24$. On the base on this and [OdS79], [Lev79] we have $N(B^8)=240$ and $N(B^{24})=196\,560$. No other precise value of $N(B^d)$ is known.

For other contexts see also [An04], [Bö03], [Bö04], [EdRS98], [De72], [EriZ01], [Had55], [Had57], [Zo98], [Hor75], [BMP05].

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3 Kepler's conjecture

Density of any arrangement of non-overlapping equal circles in the plane is at most $\pi/\sqrt{12}$ and in the densest arrangement the centers of the circles form the hexagonal lattice (incomplete proof in [Th892], corrected in [Th10]). Similar question in E^3 for the densest packing of equal balls is much more complicated.

Kepler's conjecture [Ke611]: Arrange the balls such that their centers form the hexagonal lattice in the plane. The densest packing of balls one get by translations of such planar arrangement.

Gauss [Ga831]: among the *lattice* packings the above mentioned arrangement is the densest.

So, to prove the Kepler's conjecture it is necessary to show that any *irregular* arrangement has the density less than the above mentioned regular arrangement. In this reason Kepler's conjecture became the longest unsolved problem of combinatorial geometry.

Fejes Tóth László [Fe53] showed that to obtain the maximum packing density this can be reduced to a finite though very large number of calculations.

Rogers [Ro58] showed almost sharp value of the density.

Hales [Ha92] – it is sufficient to find the extremum of the function of 150 variables.

Hsiang [Hs93]: I have a direct geometrical proof of the Kepler's conjecture.

Gábor Fejes Tóth (son of L. Fejes Tóth) in his referee report on Hsiang's paper: „As far as details are concerned, my opinion is that many of the key statements have no acceptable proofs."

Controversy between Hsiang and Hales [Ha94], [Hs95].

Hales [Ha97], [Ha97b], [Ha00] reported that his proof is complete. But his proof (based on the idea of Fejes Tóth László) had 250 pages and included a 3-gigabyte computer program, because it was necessary to analyze more than 5000 configurations of spheres.

Non-standard review, 12-membered panel: András Bezdek, Michael Bleicher, Károly Böröczky, Károly Böröczky, Jr., Aladár Heppes, Wlodek Kuperberg, Endre Makai, Attila Pór, Günter Rote, István Talata, Béla Uhrin, Zoltán Ujváry-Menyhárd. In 2003, the head of the panel of reviewers, Gábor Fejes Tóth, announced that a group of reviewers is 99% certain correctness of the proof, but - of course - cannot confirm the accuracy of all computer calculations.

Detailed proof can be found in [HaF06] and [FeH11].

Results for $d-$dimensional space, $d \geq 4$, can be found e.g. in [Var95], [CoS96], [Ro58], [Be02a], [CoE03].

4 Happy End problem

Find the smallest integer $f(n)$ such that every set of $f(n)$ points in general position in the plane contains the vertices of some convex $n-$gon.

Conjecture. (Erdős – Szekeres [ErS35]) Every set of $2^{n-2}+1$ points in general position in the plane contains a convex $n-$gon, so $f(n)=2^{n-2}+1$.

Theorem. ([ErS35]) For each integer $n \geq 4$ there is the smallest number $f(n)$ such that from any $f(n)$ points in general position in the plane one can choose n points such that they form the vertices of a convex $n-$gon.

Only estimates are known: $f(n) \leq \binom{2n-4}{n-2}+1$ (see [ErS35]) and $2^{n-2}+1 \leq f(n)$ (see [ErS60] – this lower bound is perhaps the best possible one). See [ChuG98], [KlP98] and [TóV98] for small improvements of the upper bound.

Conjecture. ([Er78]) From a sufficiently large number $G(n)$ points in general position in the plane is always possible to choose n points such that they are the vertices of an empty convex $n-$gon, so such $n-$gon which contains no other point of the given $G(n)-$point set in his interior.

Natural task is to find the smallest possible value of $G(n)$.

Trivially, $G(3)=3$, $G(4)=5$. Harborth [Har78] proved $G(5)=10$. Surprisingly, Horton [Ho83] for every n constructed such set of n points that does not contain the vertices of an empty convex $7-$gon, and thus proved the result $G(7)=\infty$.

Estimates for the value $G(6)$ are in [Va92], [Ov03], [Ni07], [Ge08], [Kos07] and generalizations in [BiF89], [BiF89b], [BK01], [PaT00], [PóV02].

5 Sylvester – Gallai theorem

Sylvester [Sy893]: „Is it true that any finite set of points in the Euclidean plane, not all on a line, has two elements whose connecting line does not pass through a third?“ Such connecting line is called ordinary line. The number of ordinary lines determined by finite noncollinear point set we denote by l_2.

Theorem. (Sylvester – Gallai) Non-collinear finite point set determines at least one ordinary line.

The most beautiful proof comes from L. M. Kelly; Kelly's proof is published in [Cox48] and also in excellent book [AiZ04] *Proofs from THE BOOK*.

Conjecture. ([Di51], [Mot51]) For every $n \neq 7,13$, the number l_2 of ordinary lines determined by n noncollinear points is at least $n/2$.

Theorem. ([KeM58]) $l_2 \geq \frac{3}{7}n$.

Hansen ([Han81]): incomplete proof of the conjecture $l_2 \geq n/2$.

Theorem. ([CsS93]). If $n \geq 8$, then $l_2 \geq \frac{6}{13}n$.

For $n=2m$ Motzkin found configurations with exactly $\frac{1}{2}n$ ordinary lines. So, the lower bound $n/2$ in the conjecture cannot be enlarged. For odd n the best known configurations (Böröczky) determine at least $\frac{3}{4}n$ ordinary lines.

Theorem. ([GrT13]) If n_0 is sufficiently large, then $l_2 \geq n/2$ for every $n \geq n_0$.

Despite the fabulous result of Green and Tao the problem is not entirely closed. In particular: how large is n_0?

The history of the solution of Sylvester's problem is a little confused and complicated, probably due to the 2nd World War. A good overview can be found in [BMP05], but for details we can recommend [Er83], [Er43], [St44], [Mel 41], [Bor83], [BorM90], [BrE48], [Cox89], [EHK63], [Ed70], [HeK60], [Ku72], [La55], [Li88], [Mot51].

Related problems have enormous literature. For further contexts and especially for many generalizations (besides other to pure combinatorial access, e.g. number of circles, horocycles, planes, unit circles, ...) see [AgA92], [Bá79], [Bá*94], [Bá98], [Bá99], [Bá06], [Bá07], [CrM68], [Brk72], [St44], [Hh55], [BrE48], [CFG94], [BáČ07], [KoP60], [Grü99], [BorM90], [Ch70], [ErP95], [PaP00], [Han65], [Han80], [Mot51], [Pu86], [BMP05], [Tu77], [SzT83], [PaT97], [Pa*04], [Sz97], [Grü72], PaS04], [Al02], [Bá90b], [Bá*95], [Bá*97], [BáK01], [Bá03b], [BáB07], [Be99], [BFT01], [Ele84], [El67], [GRS90], [Har85], [HarM86], [JaH83], [Ju70], [KlW91], [Ni05], [PPS04], [Pi02], [Pi03], [Ra30], [Sco70], [SSV05], [TaV06], [TóV05], [Bá13], …

6 Finite containers

Auerbach, Banach Mazur and Ulam proved that for every positive integer d and for every $V > 0$ there exists a number $f_d(V)$ such that each system of $d-$dimensional convex sets $M_i, i \in J$ with diameter at most 1 and with a total volume at most V can be packed into $d-$dimensional cube with side of length $f_d(V)$.

Problem. What is the smallest possible value of the number $f_d(V)$?

To answer this question it will be extremely difficult. For *parallel* packing of *boxes* the first upper estimate of the number $f_d(V)$ found Kosiński [Ko57]. This estimate improved Moon and L. Moser [MoM67]. For the dimension $d = 2$ better estimates found Meir and L. Moser [MeM68].

Question. Is it possible to pack all the rectangles $R_i = \frac{1}{i} \times \frac{1}{i+1}$, $i = 1, 2, 3, \dots$ into the unit square?

Question. Is it possible to pack all the squares with sides $\frac{1}{2i+1}$, $i = 1, 2, 3, \dots$ into the rectangle with area $\frac{1}{8}\pi^2 - 1$?

Both of previous problems are well grasp because specific systems of boxes are packed. Good estimates are in [Je94] and [Je95], better estimates in [Bá90], [Bá90b], [Bá98b] and almost sharp estimates (using a computer) in [Pau98]. But the precise answers are still not known.

Problem. ([MoM67]) Determine the least number A such that every system of squares with total area 1 can be parallely packed into some rectangle with area A.

Kleitman and Krieger [KlK70] proved that every such *finite* system can be packed into the rectangle with sides 1 and $\sqrt{3}$ and this upper estimate they improved in [KlK75] to the rectangle with sides $\frac{2}{\sqrt{3}}$ and $\sqrt{2}$; so, they proved $A \le \frac{4}{\sqrt{6}} \doteq 1{,}632\,993\,162$. Novotný [No95] showed nontrivial lower bound $A \ge \frac{2+\sqrt{3}}{3} > 1{,}244$ for the system of three squares with length of side $\frac{1}{\sqrt{6}}$ and one square with length of side $\frac{1}{\sqrt{2}}$. Because of small squares can be packed very economically, it is almost sure that exactly this configuration gives the extremum.

Conjecture. $A = \frac{2+\sqrt{3}}{3}$.

Nontrivial upper estimate of Kleitman and Krieger for the finite systems of squares improved Novotný in [No96] to $A < 1{,}53$ and in the paper [No99] he showed that for every five-element system of squares the equality $A = \frac{2+\sqrt{3}}{3}$ holds. Just this equality he showed also for 6, 7 and $8-$element systems in [No02]. This significantly strengthens the belief in the validity of the conjecture that the above mentioned 4-element system of squares is extremal. The upper estimate only recently improved (using a computer by a suitable discretization) Hougardy [Hou10] to $A \le 1{,}4$, but also this estimate is still far from the expected exact value $A = \frac{2+\sqrt{3}}{3}$.

There are many open questions in higher dimensions.

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In Malfatti [Ma803] one can found the problem of packig of three circles into the triangle such that the sum of area of packed circles is maximal. Surprisingly, this problem was solved only 191 years later, when Zalgaller and Los [ZaL94] proved that the maximum gives so called *greedy algorithm*. Sharp values for the packing of n equal circles into the equilateral triangle are known only for triangle numbers $n = \frac{j(j-1)}{2}$ ([Ol61]) and by Erdős forecasted result for $n = 14$ (see [Pay97]). Besides this there are known only some estimates reached by computer (see e. g. [GrL95]).

Exchange the triangle and circle gives the following problem.

Problem. ([AnM06]) Maximize the sum of area of three non-overlapping triangles which are packed in given circle.

The *greedy algorithm* does not give maximum for this problem. Andreescu and Mushkarov conjectured that the maximum area of $n \ge 3$ triangles packed in circle shall be achieved by dividing on triangles the regular $(n+2)$-gon inscribed in the circle. Bezdek and Fodor [BeF10] and independently also [Bá10] proved the weaker version of this conjecture: if all vertices of packed triangles are on the circle. A. Bezdek and Fodor [BeF10] noted that this problem seems *hopelessly difficult*. On the lecture we show a result reached recently by a computer.

* * *

For given integer $k \ge 2$ we are looking for the maximum radius $r = r(k) > 0$ such that k balls with radius r can be packed into the greater ball with radius $R > r > 0$.

Because of the position of the ball is precisely determined by the position of its centre, we can consider admissible arrangement of the centres of balls.

For the positive integer k denote $h(k)$ the greatest number such that into the circle with radius 1 can be packed k points such that the least distance of pairs of the packed points is $h(k)$. Sharp values of $h(k)$ for $k \le 10$ showed Pirl [Pir69]. Besides this there are known only $h(11)$ (see [Me94]) and $h(12)$, $h(13)$, $h(14)$ and $h(19)$ (see [Fo00], [Fo03b], [Fo03] and

[Fo99]). The best known lower estimates of $h(k)$ for $n \le 65$ were found in [Gr*98] using a computer.

* * *

Given integer k, find the maximum radius $r = r(k)$ such that k smaller balls with radius r can be packed into the unite cube. In the plane there are known results for $k \le 28$ (see e.g. [NuÖ99], [Sc65], [ScM65], [Pe*92], [Ma04]). In [LGS97] one can found so called *billiard* algorithm for finding dense arrangements of centres of circles. Many of them are maybe the best possible one, but the proof of optimality absents. For a good survey see [AmB00].

Problem. ([Mo66], [Gu75], [Mo91], [MoP94], [BMP05]) Denote by $f(d)$ the maximum number of points which can be placed into $d-$dimensional unit cube C^d such that all determined distances of points are at leas 1. Find sharp values of $f(d)$ at least for small values of d.

Such arrangement of points we call admissible set. Trivially, $f(d) = 2^d$ for $d = 1,2,3$. Unicity of the packing in dimension 3 was proved in [BáB01] and [Sc66]. Besides this there is known only one precise value: $f(4)$, which was shown together with unicity of the configuration in [BáB03]. No other sharp values of $f(d)$ are known (see also [Bö04]).

Theorem. ([BáB03]) If an admissible set of $f(5)$ points contains all vertices of the unit cube C^5, then $f(5) = 34$. If an admissible set of $f(6)$ points contains all vertices of the unit cube C^6, then $f(6) = 76$.

Lower bounds (see [BáB03], [BáB07], [BáB08b]): $f(5) \ge 34$, $f(6) \ge 76$, $f(7) \ge 184$, $f(8) \ge 481$, $f(9) \ge 994$, $f(10) \ge 2452$, $f(11) \ge 5464$, $f(12) \ge 14705$. In [Horv10] the author constructed by Hamming codes admissible packings of $\dfrac{3^d + 2(d-1)3^{d/2} + 1}{2d}$ points into the $d = 2^k -$dimensional unite cube.

To obtain an upper bound is much more difficult. In [BáB08] we showed the upper bounds of $f(d)$ for $d = 6, \ldots, 12$. All this upper bounds was a little improved in [Ta10].

In [BáB12b] was proved $f(6) \le 120$, and this time this is the best known result.

In [BáB07] was proved the estimate $f(5) \le 44$; by another (very complicated) method Joós in [Jó08] showed the estimate $f(5) \le 43$, and he improved this in [Jó10] to $f(5) \le 42$. In this time the best known upper estimate is $f(5) \le 40$ (see [BáB12]).

Conjecture. $f(5) = 34$, $f(6) = 76$, $f(7) = 184$, $f(8) = 481$.

Asymptotic upper estimate $f(d) \le d^{d/2}\left(1 + \frac{1}{d}\right)^d \sim d^{d/2} e^{(1+o(1))\sqrt{d}}$ for d sufficiently large ([BáB03]) was improved on the base of writing communication [Ma] and using [FeK93] and [KaL78] to $f(d) \le d^{d/2} \cdot 0{,}63091^d e^{o(d)}$ in [BáB08]. Using [Ma] was shown also nontrivial lower estimate $f(d) \ge d^{d/2} \cdot 0{,}2419707^d \, \Omega\left(\sqrt{d}\right)$ (see [BáB08]).

7 Mix

If we take n points in the plane, $n \geq 4$, then not all pairs of the points can determine the same distance. Therefore it is natural to ask ([Er46]) at most how many times occurs the same distance. Sharp values were found only for $n \leq 14$.

Because just looking problem – despite numerous attempts – is still not fixed, it offers a wide variety of different approaches (often initiated by Erdős). It was examined many special cases and related problems:

what is the minimum number of distances determined by n points in the plane ([AEP91], [Bec83], [Chu84], [ChuST92], [Ele95], [ErF96], [ErF97]);

distribution of the distances and frequent long distances ([He56], [Ve85], [Ve87], [Ve96]);

distances of points on the sphere ([ChG85], [ChK73], [EHP89]) or distances between the vertices of convex polygons ([Al63], [Al72], [Fi95], [Fi97], [Se03]);

many other problems ([BáK01], [Fi98], [Jó09], [Koj01], [Koj02], [Wei12], [La*08], [Br98], [Br98b], [Er75], [Er82], [Er84], [Er86], [BáB94], [PuS10], [Zh11]).

Only few of those partial problems were completely solved.

* * *

One of the central problems of discrete geometry is so called decomposition, i.e. to divide the body into smaller parts. Very easily it can be shown that the circle with diameter D can be divided into three parts with smaller diameter, but it cannot be divided into two parts with smaller diameters. Easily can be proved also the first part of the 3-dimensional analog, namely that the ball of diameter D can be divided into four parts with smaller diameters.

Borsuk [Bors32] proved for any $d \geq 2$ that $d-$dimensional ball with diameter D cannot be divided into d parts with smaller diameters.

Theorem. ([Bors33]) Each region in E^2 with diameter D can be divided into three parts with smaller diameters.

In the proof of previous Borsuk's theorem the key role has the following (extremely useful) old geometric theorem.

Theorem. ([Pá21]) Every planar region F with radius D can be inscribed into the regular hexagon of the width D.

Borsuk's number is the smallest number $\beta(d)$ such that every set in E^d with radius 1 can be divided into $\beta(d)$ parts with the diameter less than 1. In a consequence of [Bors32] the inequality $\beta(d) \geq d+1$ is clear.

The conjecture $\beta(d) \leq d+1$ was supported by the extension of Borsuk's theorem for dimension 3, so that each region in E^3 with diameter D can be divided into four parts with smaller diameters (first proof was very complicated and can be found in [Eg55], much simplier proofs are e.g. in [Grü57], [He57]).

Counter-example found Kahn and Kalai [KaK93] in dimension 1326. Then arises a competition broke out in search of the smallest dimension where that conjecture is not valid: 946 - Nilli [Nil94], 561 - Raigorodskij [Rai97], 560 - Weissbach [We00], 323 - Hinrichs

[Hi02], 321 - Pikhurko [Pik02], 298 – Hinrichs and Richter [HiR03]. The authors of these improvements are agreed that the wanted smallest dimension is much smaller, perhaps somewhere between 4 and 10.

Conjecture. (Soifer [So10]) There is a bounded set in E^4, which cannot be divided into 5 parts with smaller diameters.

This is encouraging hypothesis, because dimension 4 is still quite clearly visible. Less encouraging is that since 2002, still holds the Hinrichs record 298.

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All good luck and a lot of pleasant moments in solving open problems of this beautiful area of mathematics. Maybe it is sufficient to attempt the happiness by one coffee, because by Rényi *mathematician is a machine that converts coffee on theorems*.

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